MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

1.017 Computing and Data Analysis for Environmental Applications

Quiz 2 (with solutions)
Thursday, November 8, 2001

Please answer all questions on a separate piece(s) of paper with your name clearly identified:

Problem 1 (15 points)

i) Plot a sample cumulative distribution function (CDF) for the following soil conductivity (in cm/day) samples on the attached piece of graph paper.

2.72 1.44 24.28 6.96 6.40 16.30 11.72 8.63 2.94 10.52 8.48 1.63
12.77 1.50 0.14

ii) Use this sample CDF to estimate the probability that the soil conductivity at any given point lies between 5 and 10 cm/day.

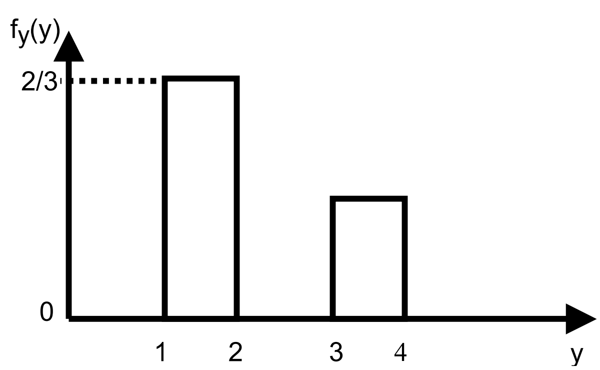
Solution:

i) See `quiz01_2sol.m` for MATLAB program that plots CDF

ii) Prob($5 < y \leq 10$) = 0.67 - .40 = 0.37

Problem 2 (15 points)

Find the mean of the following probability density function:



Solution:

$$E[y] = \int_1^2 y(2/3)dy + \int_3^4 y(1/3)dy = \left[\frac{y^2}{3}\right]_1^2 + \left[\frac{y^2}{6}\right]_3^4 = \frac{4}{3} - \frac{1}{3} + \frac{16}{6} - \frac{9}{6} = \frac{13}{6}$$

Problem 3 (20 points)

The expression $z = y/(y + y_{half})$ is frequently used to describe the relationship between algal growth rate z and nutrient concentration y. Suppose the "half saturation constant" $y_{half} = 1$ and that y is uniformly distributed between 1 and 2. Write a MATLAB function that uses a stochastic simulation approach to plot the cumulative distribution function of z. Attached are descriptions of the MATLAB functions **`rand`**, **`hist`**, and **`cdfplot`**.

Solution:

See **`quiz01_2sol.m`** for MATLAB program that plots the histogram and CDF

Problem 4 (25 points)

The position y_t of a solute particle moving in a turbulent velocity field can be described as follows:

$$y_t = y_{t-1} + \Delta v_t$$

where t is a time index (t=1,2, … n), $y_0 = 0$, Δ is a constant time step, and $v_1, v_2, \ldots v_n$ are random velocities. For example, if $\Delta = 0.5$ sec and velocity is measured in cm/sec, the position after 5 time steps is:

$y_5 = 0.5(v_1 + v_2 + v_3 + v_4 + v_5)$

This process is an example of a random walk (or Brownian motion). It is the basis for most models of solute dispersion.

Suppose that the velocities are **independent** random variables, each with mean $E[v_i] = 0$ and variance $Var[v_i] = 1$ cm/sec.

a) What are the mean and variance of the particle position y_n after n time steps?
b) Sketch the probability density function of y_{20}, clearly labeling the mean and standard deviation with appropriate numerical values.

Solution:

a) $E[y_n] = 0$, $Var[y_n] = (0.5)^2 n$
b) PDF is normal with mean $E[y_{20}] = 0$ and $SD[y_{20}] = (0.5)(20)^{0.5}$

Problem 5 (25 points)

Suppose that the streamflow in a particular river on a given day is approximately an exponentially distributed random variable with a mean (parameter *a*) of 5 m^3/sec. Write a MATLAB code that uses a stochastic simulation approach to plot the histogram and cumulative probability distribution of the **maximum** daily streamflow observed over a 100 day period. Attached are descriptions of the MATLAB functions **`exprnd`** , **`max`**, **`hist`**, and **`cdfplot`**.

Solution:

See **`quiz01_2sol.m`** for MATLAB program that plots the histogram and CDF