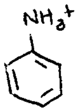
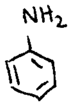
Problem set #6 2-26, 3-1, 2, 7, 9 Solutions

26.

$C_6H_5NH_3^+ \rightleftharpoons C_6H_5NH_2 + H^+$ $\quad K = 10^{-4.6}$

anilinium ion aniline





Light with wavelengths < 290 nm doesn't reach the earth's surface; this is UV light, and it's absorbed by ozone (see p. 161 figure). This means aniline will be effectively degraded by direct photolysis, since it absorbs light above 290 nm, but anilinium will not.

We want a pH where aniline predominates:

$$\frac{[C_6H_5NH_2][H^+]}{[C_6H_5NH_3^+]} = 10^{-4.6}$$

at pH 3.5, $\dfrac{[An]}{[AnH^+]} = \dfrac{10^{-4.6}}{10^{-3.5}} = 0.079$

pH 6.5, $\dfrac{[An]}{[AnH^+]} = 79$

pH 9.5, $\dfrac{[An]}{[AnH^+]} = 79{,}000$

We would not want to acidify the water, because then anilinium would predominate. Raising the pH seems reasonable, but then an extra neutralizing step would be needed, which would add to the cost. At pH 6.5 there is still significantly more aniline (also, as aniline is degraded, more anilinium will get deprotonated to maintain equilibrium), so leaving the pH unchanged would be best.

1.

$K = 5 \times 10^{-3}$ cm/s

$\dfrac{dh}{dx} = 0.02$

$n = 0.2$

a) specific discharge

$$q_0 = -K\frac{dh}{dx} = (5\times10^{-3}\text{ cm/s})(0.02) = \boxed{10^{-4}\text{ cm/s}}$$

(informal treatment of negative sign, which tells us flow is from higher head to lower head)

b) maximum velocity

$$v = \frac{q}{n} = \frac{10^{-4}\text{ cm/s}}{0.2} = \boxed{5\times10^{-4}\text{ cm/s}}$$

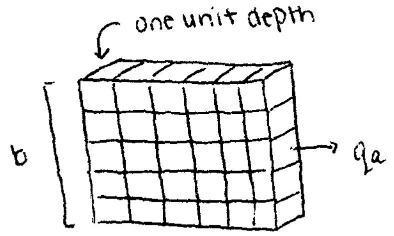
c) $b = 5m$

Transmissivity

$T = Kb = (5\times10^{-3}\,cm/s)(500\,cm) = \boxed{2.5\,cm^2/s}$

discharge per unit distance ⊥ to flow

$bq = (500\,cm)(10^{-4}\,cm/s) = \boxed{0.05\,cm^2/s}$



bq_a calculates discharge through a cross-sectional area of $b \times 1$ unit

$\boxed{2.}$ $r_w = 0.1\,m$

$Q_w = 80\,L/min$

a) find drawdown → this is steady-state, so use Theim equation

$$s_w = \frac{Q_w}{2\pi Kb}\ln\left(\frac{R}{r_w}\right)$$

$$= \frac{80\,L/min}{2\pi(5\times10^{-3}\,cm/s)(5m)}\ln\left(\frac{100\,ft.}{0.1\,m}\right) = \frac{0.08\,m^3/min}{2\pi(.003\,m/min)(5m)}\ln\left(\frac{30.48\,m}{0.1\,m}\right)$$

$$= \boxed{4.9\,m}$$

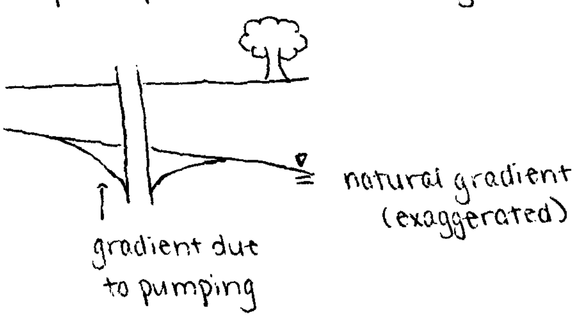
b) find hydraulic gradient due to the well:

$$\frac{ds}{dr} = \frac{Q_w}{2\pi Kbr} = \frac{0.849\,m}{r}$$

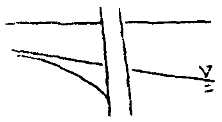
at 20 ft. $\frac{ds}{dr} = \frac{0.849\,m}{20\,ft.} \times \frac{1\,ft.}{.3048\,m} = 0.139$

at 75 ft. $\frac{ds}{dr} = \frac{0.849\,m}{75\,ft.} \times \frac{1\,ft}{.3048\,m} = 0.037$

This gradient, which is "downhill" towards the well in all directions, can be superimposed on the natural gradient.



∴ upstream.

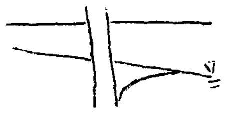


pumping-induced gradient <u>enhances</u> natural gradient

20 ft: $\frac{dh}{dx} + \frac{ds}{dr} = 0.02 + 0.139 = 0.159$

75 ft: $0.02 + 0.037 = 0.057$

downstream:

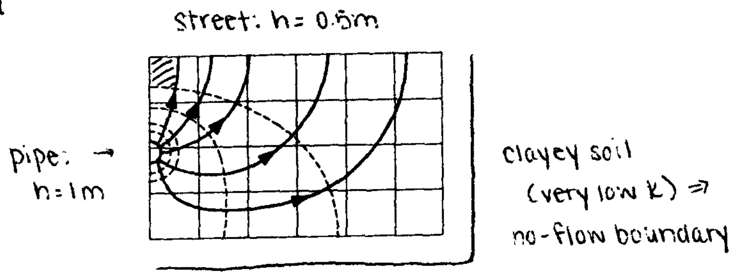


pumping-induced gradient <u>counteracts</u> natural gradient

20 ft: $\frac{dh}{dx} - \frac{ds}{dr} = 0.02 - 0.139 = -0.119$

75 ft: $0.02 - 0.037 = -0.017$

$\boxed{7.}$



$K = 10^{-3}$ cm/sec

Also note that we are only given half the flow net (i.e. 12 streamtubes total).

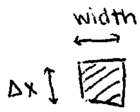
find Q per meter of pipe:

1) find specific discharge in one square (top left corner)

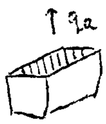
$\Delta h = \frac{1m - 0.5m}{5} = 0.1m$

$q = -K\frac{dh}{dx} = 10^{-3}\text{ cm/s} \times \frac{0.1m}{0.3m} = 3.3 \times 10^{-4}\text{ cm/s}$

$\Delta x \approx 0.3m$



2) find discharge (flow) through this streamtube



Cross-sectional area ($\perp$ to q_a) = width of square × 1 m

because we want Q "per meter of pipe"

$Q = q\,A = 3.3 \times 10^{-4}\text{ cm/s}\,(0.3m)(1m)$

$= 1\text{ cm}^3/\text{s}$

3) for total discharge, multiply by # of streamtubes (because there is an equal amount of discharge in each streamtube)

$Q_{tot} = 1\text{ cm}^3/\text{s} \times 12 = \boxed{12\text{ cm}^3/\text{s}}$ (per m of pipe)

The flow net is not very accurate because the aspect ratio is not constant -- some blocks are like squares, while others are like rectangles.

Alternative calculation (but you should understand the first one!):

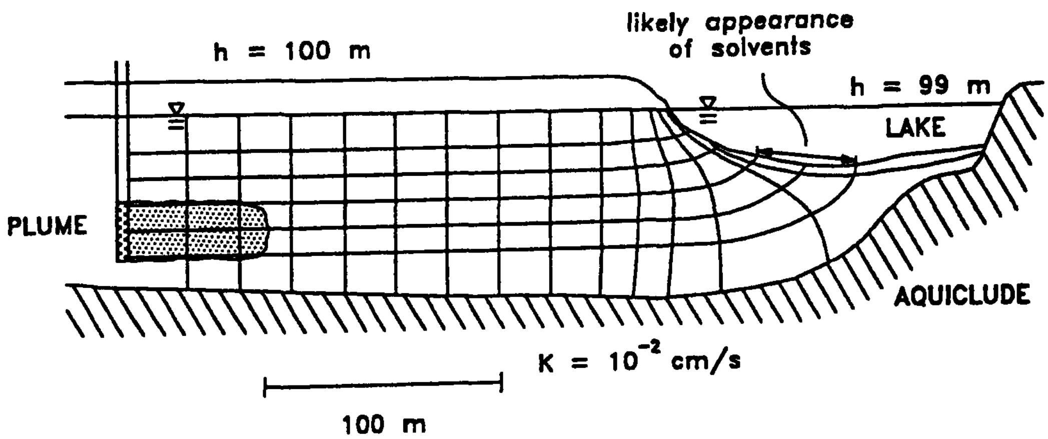
$H = 0.5\,m$ total head drop

$n_d = 5$ # head drops

$n_f = 12$ # streamtubes

$$Q_f = \frac{n_f}{n_d} K \cdot H = \frac{12}{5}(10^{-3}\,cm/s)(50\,cm) = 0.12\,cm^2/s \times \underset{\text{1 m of pipe}}{100\,cm} = 12\,cm^3/s$$

9.



-- Flow doesn't cross streamlines (note that we are neglecting diffusion + dispersion), so the solvents will appear at the lake bed wherever the appropriate streamtubes end up.

distance ≈ 300 m

$$q = -K\frac{dh}{dx} = 10^{-2}\,cm/s \times \frac{1\,m}{300\,m} = 3.3\times10^{-5}\,cm/s$$

$$v = \frac{q}{n} = \frac{3.3\times10^{-5}\,cm/s}{0.3} = 1.1\times10^{-4}\,cm/s$$

-- assume a value of n (0.2 to 0.4, p.208)

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{300\,m}{1.1\times10^{-4}\,cm/s} \times \frac{100\,cm}{m} = 2.7\times10^{8}\,sec$$

$$2.7\times10^{8}\,s \times \frac{day}{86{,}400\,s} \times \frac{yr}{365\,d} = 8.6\text{ years} \approx \boxed{9\text{ years}}$$

this is a rough approximation!

To be more precise, we could calculate the travel time across each square (if this level of precision is necessary), as done in Example 3-2.