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Homework problems on Fluid Dynamics
(1.63J/2.21J)

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oilslick.tex

1. **Oil slick on the sea** surface

Let the oil slick be modelled by an inextensible, massless and perfectly flexible film floating on the sea surface. There is no wind above but a surface wave passes beneath causing the surface and the film to have the following vertical displacement

$$\zeta = A \sin(kx - \omega t) \tag{1}$$

with $kA \ll 1$ and $\omega = \sqrt{gk}$ being th dispersion relation for deep water waves. Waves introduce a Stokes boundary layer under the film, and Eulerian streaming in the boundary layer, causing the film to drift at some mean velocity U. Find the drift velocity U by a perturbation analysis.

Suggestions:

1. Consider the water motion induced by an inextensible film having g vertical dispoacement $\zeta = (A \sin(kx - \omega t)$ and make use of the following result that a point on the film has the velocity components to the leading order

$$u(x, \zeta, t) = \frac{A^2}{4} \omega k \cos 2(kx - \omega t) \left[1 + O(kA)^2\right], \tag{2}$$

$$v(x, \zeta, t) = -\omega A \cos(kx - \omega t) \left[1 + O(kA)^2\right] \tag{3}$$

   These results are due to G.I. Taylor for another problem.

2. Assume for simplicity that the amplitude A is much smaller than the boundary layer thickness $\delta = \sqrt{2\nu/\omega}$.

3. Find the second-order Eulerian streaming velocity in the boundary layer and its limit $\bar{u}_\infty$ at the outer edge of the boundary layer.

4. The film must advance at the drift velocity $U = -\bar{u}_\infty$.