UNIT-4

G R E E N

1. *The general solution of dy/dx=* e^{x+y} is (b)

a) $e^x + e^y = c$ *b)* $e^x + e^{-y} = c$ *c)* $e^{-x} + e^y = c$ *d)* $e^{-x} + e^{-y} = c$

2. Find the differential equation corresponding to $y=ae^x + be^{2x} + ce^{3x}$ (a)

a) $y^{111} - 6y^{11} + 11y^1 - 6y = 0$ *b)* $y^{111} + y^{11} - 3y^1 = 0$ *c)* $y^{11} + 2y^1 + y = 0$ *d)* $y^{111} - 2y^{11} + 3y^1 + y = 0$

3. Find the differential equation of the family of curves $y=e^x$ (Acosx+Bsinx) (d)

a) $y^{11} - 2y^1 + 3y = 0$ *b)* $y^{11} - 3y^1 + y = 0$ *c)* $y^{11} - 2y^1 + 3y = 0$ *d) none*

4. Form the differential equation by eliminating the arbitary constant $y^{2=}(x-c)^2$ (a)

a) $(y^1)^2 = 1$ *b)* $y^{11} + 2y^1 = 2$ *c)* $(y^1)^2 = 0$ *d) none*

5. Find the differential equation of thefamily of parabolas having vertex at the origin and foci on y-axis (b)

a) $xy^1 = 2x$ *b)* $xy^1 = 2y$ *c)* $xy^1 = 4y$ *d) none*

6. Form the differential equation by eliminating the arbitary constant tanxtany=c (b)

a) $y_1 (\tan y + \sec^2 x) = 0$ *b)* $y_1 (\tan y \sec^2 y) + \tan y \sec^2 x = 0$ *c)* $y_1(\tan x \sec^2 x) + \tan y \sec^2 x = 0$ *d)none*

7. Obtain the differential equation of the family of ellipse $x^2/a^2 + y^2/b^2 = 1$ (c)

a) $xyy^{11} + xy^1 = 0$ *b)* $xy^{11} + xy = 0$ *c)* $xyy^{11} + x(y^1)^2 - yy^1 = 0$ *d) none*

8. The family of straight lines passing through the origin is represented by the differential equation (b)

a) ydx + xdy=0 b) xdy – ydx=0 c) xdx + ydx=0 d) ydy – xdx=0

9. The order of $x^3\, d^3y/dx^3 - 3y = x$ is (c)

a) 2 b)3 c) 1 d) none

10. The degree of the differential equation $[d^2y/dx^2 + (dy/dx)]^{3/2} = a\, d^2y/dx^2$ is (a)

a) 2 b)3 c)1 d) none

11. The family of the straight lines passing through the origin is represented by the differential equation

(b)

a) ydx + xdy=0 b) xdy – ydy=0 c) xdx + ydx=0 d) ydx – xdx=0

12. The solution of the differential equation dy/dx+y/x=x^2 under the condition that y=1 when x=1 is

(b)

a) 4xy = x^3+3 b) 4xy = x^4 +3 c) 4xy = y^4+3 d) 4xy = y^3 + 3

13. The equation dy/dx + ax+hy+g/hx+by+f=0 is

(c)

a) Homogenous b) variable seperable c) exact d) none

14. The rate at which bacteria multiply is proportional to the instantaneous number present. If the original number Doubles in 2 hr, in how many hours will it triple?

(C)

(a) $\frac{2\ log\ 2}{log\ 3}$*hrs*

(b) $\frac{log\ 2}{log\ 3}$ *hrs*

(c) $\frac{2\ log\ 3}{log\ 2}$*hrs*

(d) $\frac{log\ 3}{log\ 2}$ *hrs*

15. The general solution of law of natural decay is

(d)

(a) y(t)=y(0) + ce^{-kt}
(b) y(t)=ce^{kt}
(c) y(t)=y(0) + ce^{kt}
(d) y(t)=ce^{-kt}

16. The temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself is known as (c)
(a) heat flow
(b) law of netural decay
(c) newton law of cooling
(d) law of natural growth

17.

(d)

If f(D)=D^2-2, $\frac{1}{f(D)}e^{2x}$

(a) $-e^{2x}$

(b) $\frac{-e^{2x}}{2}$

(c) e^{2x}

(d) $\frac{e^{2x}}{2}$

18.

(c)

The C.F. of $D^2(D-1)^3(D+1)y=e^x$ is

(a) $(c_1+c_2)e^{-x}+(c_3+c_4x+c_5x^2)e^{-x}+c_6e^x$

(b) $(c_1+c_2x)+(c_3+c_4x+c_5x^2)e^x+c_6e^{-x}$

(c) $c_1+c_2x+(c_3+c_4x^2+c_5x^3)e^x+c_6e^{-x}$

(d) $(c_1+c_2)+(c_3+c_4x+c_5x^2)e^{-x}+c_6e^x$

19. The differential equation of orthogonal trajectories of the family of curves $y^2=4ax$, where 'a' is the parameter is (d)

a) y dy/dx=-2x b) ydy/dx=2x c)xdy/dx=2y d) none

20. The differential equation of the orthogonal trajectories of the family of curves $xy=a^2$ where a is the parameter is (c)

a) y dy/dx=x b) y dy/dx+x=0 c)$y^2-x^2=2c$ *d) none*

21. The differential equation of the orthogonal trajectories of the family of curves r=aθ where 'a'is the parameter is (b)

a) a log r/θ=c b) a log r+θ=c c) a log r-θ=c d) a log rθ=c

22.

(a)

The P.I. of (D^2+9)y=cos3x is

(a) $\frac{x\sin 3x}{6}$

(b) $\frac{-x\sin 3x}{6}$

(c) $\frac{-x\cos 3x}{6}$

(d) $\frac{x\cos 3x}{6}$

23. The general equation of $(D^2-4D+3)y=\cos 2x$ is (a)

a) $C_1e^x+c_2e^{3x}-1/65(\cos 2x+8\sin 2x)$ b) $C_1e^x-c_2e^{3x}-1/65(\cos 2x+8\sin 2x)$ c) $C_1e^x+c_2e^{3x}-1/65(\cos 2x-8\sin 2x)$ d) $C_1e^x-c_2e^{3x}-1/65(\cos 2x-8\sin 2x)$

24. The C.F of $y''-2y'+2y=0$ (b)

a) $e^x(C_1\cos x-C_2\sin x)$ b) $e^x(C_1\cos x+C_2\sin x)$ c) $e^x(C_1\cos 2x+C_2\sin x)$ d) $e^{-x}(C_1\cos x+C_2\sin x)$

25. The C.F of $(D^3+4D)y=0$ (c)

a) $C_1+C_2\cos x+C_3\sin x$ b) $C_1-C_2\cos 2x+C_3\sin x$ c) $C_1+C_2\cos 2x+C_3\sin 2x$ d) $C_1-C_2\cos 2x-C_3\sin 2x$

Y E L L O W

1. The P.I of $(D^2-5D+6)y=e^{2x}$ is (a)
a) $-x\,e^{2x}$ *b)* $x\,e^{2x}$ *c)* e^{2x} *d)* 0

2. P.I of $(D+1)^2y=x$ is (b)
a) x *b)* $x-2$ *c)* $(x+1)^2$ *d)* $(x+2)^2$

3. $1/D^2+D+1(\sin x)=$ (b)
a) $\sin x$ *b)* $-\cos x$ *c)* $1/3\,\sin x$ *d)* $1-\cos x$

4. P.I of $(D-1)^4y=e^x$ is (a)
a) $x^4/4!(e^x)$ *b)* x^4e^x *c)* e^x *d)* $e^x/4$

5. The value of $1/D-2(\sin x)$ is (d)
a) $-1/5(\cos x+\sin x)$ *b)* $1/5(\cos x)$ *c)* $1/5(\sin x)$ *d)* $1/5(\cos x-2\sin x)$

6. The value of $1/D^2+4(\sin 2x)$ is (d)
a) $1/5\,(\sin 2x)$ *b)* $-1/5\,\sin^2x$ *c)* $1/5(\cos 2x)$ *d)* $-1/4\,\cos 2x$

7. $1/D^2-1(e^x)=$ (d)
a) $1/2(xe^x)$ *b)* $-1/2(xe^x)$ *c)* $x^2/2(e^x)$ *d)* none

8. $1/D+2(x+e^x)=$ (d)
a) $-x/4-1/16+e^x/3$ *b)* $x/4+1/16-e^x/3$ *c)* $x/4-1/16+e^x$ *d)* none

9. P.I of $(D^4-1)y=e^x\cos x$ (b)

a) $-e^x\cos y/6$ b) $-e^x\cos x/5$ c) $-e^x\cos x/3$ d) $e^x\cos x/5$

10 .C.F of $(D-1)^2y=\sin 2x$ is (a)
a) $(c_1+c_2x)e^x$ *b)* $(c_1+c_2x)e^{-2x}$ *c)* $c_1x+c_2e^x$ *d)* none

11. P.I of $(D^2+1)y=x^2e^{3x}$ is (c)

a) $e^{3x}/250\,(25x^2+30x+30)$ b) $e^{3x}/250\,(25x^2-30x-30)$ c) $e^{3x}/250\,(25x^2-30x+30)$ (d) $e^{3x}/25\,(25x^2-30x+30)$

12. P.I of $(D^2-2D+1)y=\cosh x$ is (a)
a) $x^2/4(e^x)+e^{-x}/8$ *b)* $x^2/4(e^{-x})+e^x/8$ *c)* $x^2/4(e^x)$ *d)* $c_1e^x+c_2e^{-x}$

13. If 30% of the ratio active substance disappears in 10days ,how long will it take for 90% of it to disappear (b)
a) 34.5days b) 64.5days c)100.0days d)55.5days

14. a bacterial culture, growing exponentially, increases from 100 to 400 gms in 10 hrs. How much was present after 3 hrs from initial instant (a)

a) 141.4gms b) 141.3gms c) 141.2gms d) 141.1gms

15. The number N of bacteria ina culture grew at a rate proportional to N. The value of N was initially 100and increased to 332 in one hour. What was the value of N after 1(1/2) (c)

a)600 b)610 c)605 d) none

16 .Abody is originally at 80'c and cools down to 60'c in 20 minutes. If the temperature of the air is 40'c ,find the temperature of the body after 40 minutes (c)

a)20'c b)40'c c)50'c d)60'c

17. Bernoullis equation is of the form (a)

a) $dy/dx+py=Qy^n$ b) $dy/dx+Qy=Qy^n$ c) $dy/dx+py=py^n$ d) $dy/dx+ab=0$

1 8. P.I of $(D^2-2D+4)y=e^{2x}\cos x$ is (b)

a) $e^{2x}(2\sin x+3\cos x)/3$ b) $e^{2x}(2\sin x+3\cos x)/13$ c) $e^{2x}(2\sin x+\cos x)/13$ d) $e^{2x}(2\sin x-3\cos x)/13$

19. find the equation of orthogonal trajectories of circles r=acosθ (a)

a) r=c sinθ b)r=c cosθ c)r=c secθ d) c taθx

20. (a)

V is function of x, $\frac{1}{f(D)}$ xV=

(a) $\left[x-\frac{1}{f(D)}f^1(D)\right]\frac{1}{f(D)}V$

(b) $\left[x-\frac{1}{f(D)}\right]\frac{1}{f(D)}V$

(c) $\left[x+\frac{1}{f(D)}f^1(D)\right]\frac{1}{f(D)}V$

(d) $\left[x+\frac{1}{f(D)}\right]\frac{f'(D)}{f(D)}V$

21. P.I of $(D^2+D+1)y=x^3$ (d)

a) x^3+3x^2+6 b) x^2-3x+6 c) x^3-3x^2-6 d) x^3-3x^2+6

22. The general equation of $(4D^2-4D+1)y=0$ is (d)

a) $y=c_1 e^{-x/2}+c_2 e^{-x/2}$ b) $y=(c_1x+c_2)e^{-x/2}$ c) $y=c_1e^{-x/2}+c_2e^{x/2}$ d) $y=(c_1+c_2x)e^{x/2}$

23. The C.F of $(D+1)(D-2)^2y=e^{3x}$ is (b)

a) $(c_1+c_2x)e^{-x}+c_3 e^{3x}$ b) $(c_1+c_2x)e^{2x}+c_3e^{-x}$ c) $c_1e^{-x}+c_2e^{2x}$ d) none

24. P.I of $d^3y/dx^3+y=e^{-x}$ is (a)

a) $x\,e^{-x}$ *b)* $e^{-x/3}$ *c)* $-xe^{-x/3}$ *d) none*

25. The P.I of $(D^2+a^2)y=\cos ax$ is (b)

a) $-x/2a\ \cos ax$ *b)* $x/2a\ \sin ax$ *c)* $x\cos ax$ *d)* $x\ \sin ax$

R E D

1. The population of a country increases at the rate proportional to the current population. If the population doubles in 40 years when it will be tripled (in years) (d)

(a) 10 log3/log2 b) 20 log3/log2 c) 30 log3/log2 d) 40 log3/log2

2. If the differential equation of the given family is $r=\theta\, dr/d\theta$, then the differential equation of the orthogonal trajectories is (b)

a) $dr/r=\theta d\theta$ b) $dr/r=-\theta d\theta$ c) $d\theta/\theta=\theta dr$ d) none

3. the P.I. of $e^{2x}/(D^2-6D+6)=$ (c)

a) $e^{2x}/2$ b) $e^{-2x}/2$ c) $-e^{2x}/2$ d) $xe^{2x}/2$

4. *(b)*

$$\frac{x^2+x}{(D^2-1)}=$$

(a) x^2+x+2 (b) $-(x^2+x+2)$ (c) x^2-x+2 (d) x^2+x-2

5. (c)

The solution of the differential equation is $(D^2-8D+16)y=0$ is

(a) $c_1e^x+c_2e^{-6x}$ (b) $c_1e^x+c_2e^{-3x}$ (c) $(c_1+c_2x)e^{4x}$ (d) $(c_1+c_2x)e^{2x}$

6. (b)

To find the orthogonal trajectories of the family of curves represented by the differential equation $\frac{dy}{dx}=\frac{-y}{x}$ We need to solve the differential equation

a) xdx-ydy=c b) ydy-xdx=0 c) ydy-xdx=c d) $xdx-ydy=x^2$

7. (b)

Integrating factor for solving the linear differential equation $(x+1)\frac{dy}{dx} - y = e^x(x+1)$ is

a) $e^{\frac{1}{x+1}}$ b) $\frac{1}{x+1}$ c) e^x d) e^{x+1}

8. The orthogonal trajectories of $xy = c$ is (b)

(a) $x^2 + y^2 = a^2$ (b) $x^2 - y^2 = a^2$ (c) $x^2 + 2x = c$ (d) $y^2 - 2x = c$

9. The nature of the differential equation $y \sin 2x dx - (y^2 + \cos^2 x) dy = 0$ (d)

(a) Homogeneous (b) Linear (c) Bernoulli (d) Exact

10. The differential equation of orthogonal trajectories of $ay^2 = x$ (c)
a) y dy = x dx b) 2y dy = 3x dx c) 3y dy = - 2x dx d) y dy = -2 x dx

11.The Orthogonal trajectory of the circles $x^2+(y-c)^2 = c^2$ is (a)

a)$x^2 + y^2 = cx$ b) $x^2 + y^2 = c$ C) $x^2 - y^2 = cx$ d) $x^2 + y^2 = y$

12. The orthogonal trajectories of $e^x + e^{-y} = c$ is (b)

a) $e^y + e^{-x} = k$ b) $e^y - e^{-x} = k$ c) $e^y + e^x = k$ d) $e^{-y} + e^{-x} = k$

13.In Othogonal trajectories $\frac{dy}{dx}$ is replaced by (d)

a)$-\frac{dy}{dx}$ b) $y^2\frac{dy}{dx}$ c)$-y\frac{dx}{dy}$ d)$-\frac{dx}{dy}$

14.The P.I of $(D^3+2D^2+D)y = e^{2x}$ is (c)

a)$\frac{e^{2x}}{3}$ b) $\frac{e^{2x}}{9}$ c) $\frac{e^{2x}}{18}$ d) $\frac{e^{2x}}{27}$

15.The newton's law of cooling is (a)

a)$\frac{d\theta}{dt}\alpha\ (\theta - \theta_0)$ b) $\frac{d\theta}{dt}\alpha - (\theta - \theta_0)$ c) $\frac{dt}{d\theta}\alpha\ (t - \theta_0)$ d)none

16.If money is invested at 5%, compounded continuously, in ------- yrs will the money doubled in value a)12.9 b)13.9 c)14.9 d)15.9 (b)

17.If the roots are -1,-1,2,2 then C.F is (a) a)$(c_1 + c_2x)e^{-x} + (c_3 + c_4x)e^{2x}$ b))$(c_1 - c_2x)e^{-x} + (c_3 + c_4x)e^{2x}$ c))$(c_1 + c_2x)e^{-x} + (c_3 - c_4x)e^{2x}$ d))$(c_1 + c_2x)e^{x} + (c_3 + c_4x)e^{-2x}$

18.$\frac{1}{D^3}$(cosx) (b)

a)sinx b)-sinx c)cosx d)-cosx

19.C.F of (D^3-5D^2+8D-4)y = e^{2x} is (a)

a) $c_1e^x + (c_2 + c_3x)e^{2x}$ b) $c_1e^x - (c_2 + c_3x)e^{2x}$ c) $c_1e^{-x} + (c_2 + c_3x)e^{-2x}$ d) $(c_2 + c_3x)e^{2x}$

20.The integrating factor of $\frac{dy}{dx} + \frac{y}{xlogx} = \frac{sin2x}{logx}$ (c)

a)x b)-x c)logx d)-logx

21. The integrating factor of dr+(2rcotθ+sin2θ)dθ=0 (b)

a)sinθ b)sin²θ c)cosθ d)cos²θ

22. The integrating factor of $\frac{dy}{dx} + y = e^{e^x}$ is (b)

a) e^{-x} b) e^{x} c) e^{-2x} d) e^{2x}

23. Solution of $(x^2 + y^2)dx = 2xydy$ (c)

a)$x^2 + y^2 = cx$ b) $x^2 + y^2 = c$ C) $x^2 - y^2 = cx$ d) $x^2 + y^2 = y$

24. In Othogonal trajectories $\frac{dr}{d\theta}$ is replaced by (c)

a)$-\frac{dr}{d\theta}$ b) $r^2\frac{dr}{d\theta}$ c)$-r^2\frac{d\theta}{dr}$ d)$-r\frac{d\theta}{dr}$

25.$\frac{1}{D^2+D+1}$ cosx is (a)

a)sinx b)-sinx c)xsinx d)-xsinx

R E D

1.The sum of eigen values of A are (1,-1,2) then the eigen values of Adj A are ( )

a) (-2,2,-1) b) (1,1,-2) c) (1,-1,1/2) d) (-1,1,4)

2.Rank of quadratic form $2x_1x_2+6x_1x_3-4x_2x_3$ ( )

a) 1 b) 2 c) 3 d) 0

3.The matrix $A^T.A=$ ( )

a) I b) 1 c) 0 d) 2

4. The diagonal matrix whose leading diagonal elements are equal is called a _____( )

a)unit matrix b)square matrix c)scalar matrix d)null matrix

5. $(KA)^1=KA^1$ where k is a _______ ( )

a) vector b) scalar c) zero d) one

6. tr(A+B)= ( )

a) tr A + tr B b) tr A - tr B c)tr A / tr B d)tr A * tr B

7.If A is a square matrix such that $A^2=I$ is called ( )

a)voluntary b)involuntary c) idempotent d) nilpotent

8.The matrix $A=\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ is ( )

a) hermitian b)skew-hermitian c)symentric d)none of above

10.If I is a unit matrix of order n then |I|=_______ ( )

a) 1 b) 0 c) 3 d) 5

11. The characterstic polynomial of $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ is ( )

a) $\lambda^3+7\lambda^2-16\lambda+12$
b) $\lambda^3-7\lambda^2+16\lambda-12$
c) $\lambda^3-6\lambda^2+15\lambda-9$
d) $\lambda^3+7\lambda^2-16\lambda/12$

12.The sum and product of the eigen values $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ & & \end{bmatrix}$ is ( )

1 0 2

a) 6 , 6 b) 7,9 c) 5,4 d) 6, 0

13.If λ is an eigen values of a square matrix A,then the eigen values of the matrix $(KA)^T$ when k=0 is ( )

a) λ/k b) k/λ c) $k\lambda$ d) None

14.If the order of matrix A is m x p and the order of B is p x n then the of the AB is =

( )

a) n x p b) m x p c) m x n d) n x m

15)If A & B are the matrices ,then which of the following is true ( )

a) $A+B\neq B+A$ b) $(A^T)^T \neq A$ c) $AB\neq BA$ d) all the above

15.What is A, if $B=\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ is a singular matrix ( )

a)5 b) 6 c) 7 d)8

16. If $A=\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then |A|=? ( )

a) 2 b) 3 c) 4 d)5

17. $(AB)^T$= ( )

a) B^TA^T b) A^TB^T c) AB d) BA

18. The matrix $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ( )

a) scalar b) identity c) even d)odd

19.The no of non –zero rows in an echlon form is called ( )

a) reduced echlon form b)rank of matrix

c)conjugate of the matrix d)cofactor of matrix

20.Two matrices are said to be equivalent if ( )

a) they are of the same size and have the same elements

b)one is sub matrix of other

c)there are of same size of same rank

d)Their ranks are of same

21.a square matrix $A=a_{ij}$ is a upper triangular if ( )

a) $a_{ij}=0$ for $i>j$ b) $a_{ij}=0$ for $i=j$ c) $a_{ij}=0$ for $i<j$ d) $a_{ij}=0$ for $i>j$

22. The rank of $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is ( )

a) 0 b) 2 c) 1 d) 3

2 3.The eigen values of unit matrix of order 3 are ( )

a) 0,0,1 b) 0,1,1 c) 1,1,1 d) 0,-1,1

24.If one of the eigen value of square matrix A is zero then the matrix is ( )

a)singular b)non-singular c)symmentric d)skewsymmentric

25.The quadratic form associated with symmentric matrix $\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ is ( )

a) $x^2-y^2+z^2-2xy-2xz+4yz$ b) $x^2-y^2+z^2+2xy-2xz+4yz$
c) $x^2+y^2+z^2-2xy-2xz+4yz$ d) $x^2-y^2+z^2-2xy-2xz+4yz$

KEY:

1) c 2) c 3) a 4) c 5) b 6) a 7) b 8) c 9) a 10) a 11)b 12)b 13) c 14) a 15) c 16) b 17) b 18) d 19) a 20) a 21)b 22)a 23) b 24)a 25)a

Y E L L O W

1.A vector over a real numbers is called _____ ..& the vectorcomplex number is called______ ( )

a) real ,complex b) comple , real c) real ,imaginary d) imaginary ,real

2. Trivial solution is also called as ( )

a) one solution b) infinity solution c) zero solution d) two's solution

3. The solution of a linear system of equations can be found out by numerical methods known as _______ ( )

a)direct method b)iterative method c) both a & b d) none of these

4.A square matrix A is symmentric if ( )

a) $A^T=-A$ b)$AA^{-1}=I$ c) $A^T=A$ d) $AA^T=I$

5.If A & B are skew-symmentric matrix then A+B is ( )

a) orthogonal b)Unitary c) Skew-symmentric d)Symmentric

6.If A&B are matrices and if AB is defined then the rank of AB is equal to ( )

a)rank of A b)rank of B c)≤min {rank A , rank B} d)≤max {rankA,rankB}

7.($A-\lambda I$)is called ………. ( )

a)singular matrix b)non singular matrix c)charecterstic matrix d)proper matrix

8.The eigen values of hermition matrix are …………….. ( )

a)purely b)regid c)real d)imaginary

9.Diagonalise of a matrix D^n =………. ( )

a)PAP b)P^TAP c)P^TA^nP d)$P^{-1}A^nP$

10.The eigen values of skew hermistion matrix are,,, ( )

a)0 b)1 c)-1 d)real

11.if A and B are 3 x 4 matrix ,then the rank of (A+B) IS ( )

a)4 b)≤3 c)≥0 d)none

12.A non zero matrix A is said to be …………. ( )

a)$A^n=0$ b)$A^n= -1$ c)$A^n=1$ d)none

13.if the matrix A = $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is …………. ( )

a)daignolised b)not daignolisable c)imaginary d)elimantary

14.sum of the charectarstics roots of matrix A=to the ( )

a)sum of the principal daignol elements of A

b)place of the matrix A

c)both A and B

d)non of above

15.if λ is an eigen value of A then K+λ(K≠0)the eigen value of the matrix ( )

a)k+AI b)A+Ki c)A+λk d)A^{-1} +KI

16.The trace of a square matrix A is equal to ( )

a) sum of eigen values b)product of eigen values c) |A| d)none of these

17.Any set of vectors which include the zero vector is ( )

a)linear independent b)linearly dependant c)cannot be linealy dependant d)none

18. If λi,i=1,2,3…,n are the eigen values of A , then the values of the matrix $(A-\lambda I)^2$are ( )

a) 0 b) $(\lambda-\lambda)^2$,i=1,2,3..n c) $(\lambda-\lambda_i)$,i=1,2,3..n d) none

19.The quadratic form corresponding to the symmentric matrix ( )

a) $x^2+4xy-4y^2$ b) $x^2-4xy+4y^2$ c) $x^2-2xy+4y^2$ d) $x^2+2xy-2y^2$

20)The modulus of each characteristics root of a unitary matrix is ( )

a) 0 b) 1 c) 2 d) 3

21.the quadratic form is +ve definite when ( )

a) all the eigen values are ≥0 and atleast one eigen value is zero

b)all eigen values are +ve

c)some eigen values are +ve

d)none of above

22. If the eigen values of are 0,0,6 then the rank of quadratic form is ( )

a) 1 b)2 c)3 d) 0

23.The index and signature of quadratic form $5x^2+2y^2+2z^2+6yz$ are ( )

a) 2 ,1 b) 3 ,1 c) 3 ,2 d) 3,3

24.If A is a hermitian then iA is ( )

a) hermitian b)skew-hermitian c) skew-symmentric d) null matrix

25.The matrix $U=\frac{1}{2}\begin{pmatrix} 1+i & -1+i \\ 1+i & 1-i \end{pmatrix}$ is ( )

a)nilponent b)orthogonal c)hermitian d)unitary

KEY:

1)a 2)c 3) c 4)c 5)c 6)c 7)c 8)a 9)d 10) b 11)a 12) c 13)d 14) a 15)b 16) b 17) a 18)b 19)a 20) b 21)b 22)a 23)a 25)b 25) d

G R E E N

1.If A&B are two invertible square matrix ,then the eigen values of AB and BA are ( )

a) equal b) different c) not determined d) none

2. If 1,2,3 are the eigen values of a square matrix A ,then the eigen values of the square matrix adj.A are ( )

a) 1,1/2,1/3 b)1,4,9 c) 6, 3, 2 d) 9,3,6

3.If the trace of 2x2 matrix A is 5 and determinant is 4 ,then the eigen values of A ( )

a) 2 ,2 b) -2, 2 c) -1 ,-4 d) 1 , 4

4. X+Y+W=0; Y+Z=0; X+Y+Z+W=0; X+Y+2Z=0 rank of the matrix is ( )

a) 2 b) 3 c) 4 d) 0

5. A square matrix A of order n x n is sometimes called as a ( )

a) rowed matrix b) n-rowed matrix c) column matrix d)n-column

6.If A,B are two matrices of the same type (order)then A+(-B) is taken has ( )

a) A-(-B) b) A=B c) A-B d) A+B

7.In the product AB the matrix A is called ______ and B is called _______ ( )

a)pre-dominant, post dominant b)pre-factor , post factor

c)pre-matrix ,post matrix d) positive , negative

8.If A is a square matrix such that A^2=I then A is called ( )

a) unitary b)Idempotent c) voluntary d)involuntary

9.If A is a orthogonal then |A| = ( )

a) 1 b) -1 c)±1 d) 0

10.If A^Θ&B^Θ be the transpose conjugates of A&B respectively then $(A\pm B)^\Theta$ ( )

a) $A^\Theta \pm B^\Theta$ b) $A^\Theta + B^\Theta$ c) $A^\Theta - B^\Theta$ d) $A^\Theta * B^\Theta$

11.Rank of aunit matrix of order 4 is …. ( )

a)2 b)3 c)1 d)4

12.The matrix A= $\begin{pmatrix} a+ic & -b+id \\ b+id & a-ic \end{pmatrix}$ is unitary if and only if $a^2+b^2+c^2+d^2$= ( )

a)0 b)1 c) – 1 d) i

13.if A= $\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$ then(A-2I)(A-3I) = ( )

a)0 b)1 c) $\begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

14.a sqare matrix A its transpose A^T have the same … ( )

a)eigen values b)eigens transpose c)eigen vector d)polynomial

15.If the eigen values of A are different then they are ( )

a)linear b)non linear c)equal d)orthogonal

16If A=. $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ then the nature of the quadratic form X^TAX ( )

a)positive definite b)positive semi definite c)negative definite d)in definite

17.if the eigen values of A or $1, 3+\sqrt{8}, 3-\sqrt{8}$ then the index and signature of the quadratic form X^TAX are ( )

a) 1,2 b) 3, 1 c) 3,2 d) 3,3

18.If A = $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ find A^{50} ( )

a) $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 0 & 3^{50} \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 \\ 50^3 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

19.The quadratic form corresponding to the symmentric matrix ( )

a) $x^2+4xy-4y^2$ b) $x^2-4xy+4y^2$ c) $x^2-2xy+4y^2$ d) $x^2+2xy-2y^2$

20.$3x^2+5xy-2y^2$ is a ____________ in two varabels X & Y ( )

a)conical form b)quadratic form c)real form d)none

21.The latent roots of $\begin{pmatrix} a & h & g \\ o & b & o \\ o & o & c \end{pmatrix}$ are ( )

a) a,b,c b)1/a,1/b,1/c c) h, g, o d)b,g,o

22.caylehmilton thermo states that every square matri satisfies its own ( )

a)characteristic polynomial b)characteristic of eqation c)none

23.if the trace of 2x2 matrix A is 5 and the determined is 4 ,then the eigen values of A are ( )

a)2,2 b)-2,2 c)-1,4 d)1,4

24.The eigen values of an idopodent matrix are ( )

a)0 only b)0and 1 only c)0 and -1 only d)-1 and 1 only

25.To find the inverse of the matrix using columns operation only we should proceed as follows A =….. ( )

a)AI_m b)AI_n c)AI d)none

KEY:

1)a 2)c 3) d 4)a 5)d 6) d 7) b 8)d 9)c 10) a 11)d 12)b 13) a 14) c 15) b 16)a 17)d 18)b 19)a 20)b 21)a 22)b 23) d 24)b 25)b

R E D

1.The sum of eigen values of A are (1,-1,2) then the eigen values of Adj A are ( )

a) (-2,2,-1) b) (1,1,-2) c) (1,-1,1/2) d) (-1,1,4)

2.Rank of quadratic form $2x_1x_2+6x_1x_3-4x_2x_3$ ( )

a) 1 b) 2 c) 3 d) 0

3.The matrix $A^T.A=$ ( )

a) I b) 1 c) 0 d) 2

4. The diagonal matrix whose leading diagonal elements are equal is called a _____( )

a)unit matrix b)square matrix c)scalar matrix d)null matrix

5. $(KA)^1=KA^1$ where k is a _______ ( )

a) vector b) scalar c) zero d) one

6. tr(A+B)= ( )

a) tr A + tr B b) tr A - tr B c)tr A / tr B d)tr A * tr B

7.If A is a square matrix such that $A^2=I$ is called ( )

a)voluntary b)involuntary c) idempotent d) nilpotent

8.The matrix $A=\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ is ( )

a) hermitian b)skew-hermitian c)symentric d)none of above

10.If I is a unit matrix of order n then |I|=_______ ( )

a) 1 b) 0 c) 3 d) 5

11. The characterstic polynomial of $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ is ( )

a) $\lambda^3+7\lambda^2-16\lambda+12$
b) $\lambda^3-7\lambda^2+16\lambda-12$
c) $\lambda^3-6\lambda^2+15\lambda-9$
d) $\lambda^3+7\lambda^2-16\lambda/12$

12.The sum and product of the eigen values $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ & & \end{bmatrix}$ is ( )

1 0 2

a) 6 , 6 b) 7,9 c) 5,4 d) 6, 0

13.If λ is an eigen values of a square matrix A,then the eigen values of the matrix $(KA)^T$ when k=0 is ( )

a) λ/k b) k/λ c) $k\lambda$ d) None

14.If the order of matrix A is m x p and the order of B is p x n then the of the AB is =

( )

a) n x p b) m x p c) m x n d) n x m

15)If A & B are the matrices ,then which of the following is true ( )

a) $A+B\neq B+A$ b) $(A^T)^T \neq A$ c) $AB\neq BA$ d) all the above

15.What is A, if $B=\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ is a singular matrix ( )

a)5 b) 6 c) 7 d)8

16. If $A=\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then |A|=? ( )

a) 2 b) 3 c) 4 d)5

17. $(AB)^T$= ( )

a) B^TA^T b) A^TB^T c) AB d) BA

18. The matrix $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ( )

a) scalar b) identity c) even d)odd

19.The no of non –zero rows in an echlon form is called ( )

a) reduced echlon form b)rank of matrix

c)conjugate of the matrix d)cofactor of matrix

20.Two matrices are said to be equivalent if ( )

a) they are of the same size and have the same elements

b)one is sub matrix of other

c)there are of same size of same rank

d)Their ranks are of same

21.a square matrix A=a_{ij} is a upper triangular if ( )

a) a_{ij}=0 for i>j b) a_{ij}=0 for i=j c) a_{ij}=0 for i<j d) a_{ij}=0 for i>j

22. The rank of $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is ( )

a) 0 b) 2 c) 1 d) 3

2 3.The eigen values of unit matrix of order 3 are ( )

a) 0,0,1 b) 0,1,1 c) 1,1,1 d) 0,-1,1

24.If one of the eigen value of square matrix A is zero then the matrix is ( )

a)singular b)non-singular c)symmentric d)skewsymmentric

25.The quadratic form associated with symmentric matrix $\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ is ( )

a) $x^2-y^2+z^2-2xy-2xz+4yz$ b)$x^2-y^2+z^2+2xy-2xz+4yz$
c)$x^2+y^2+z^2-2xy-2xz+4yz$ d)$x^2-y^2+z^2-2xy-2xz+4yz$

KEY:

1) c 2) c 3) a 4) c 5) b 6) a 7) b 8) c 9) a 10) a 11)b
12)b 13) c 14) a 15) c 16) b 17) b 18) d 19) a 20) a 21)b
22)a 23) b 24)a 25)a

Y E L L O W

1.A vector over a real numbers is called _____ ..& the vectorcomplex number is called______ (   )

a) real ,complex b) comple , real c) real ,imaginary d) imaginary ,real

2. Trivial solution is also called as (   )

a) one solution b) infinity solution c) zero solution d) two's solution

3. The solution of a linear system of equations can be found out by numerical methods known as _______ (   )

a)direct method b)iterative method c) both a & b d) none of these

4.A square matrix A is symmentric if (   )

a) $A^T=-A$ b)$AA^{-1}=I$ c) $A^T=A$ d) $AA^T=I$

5.If A & B are skew-symmentric matrix then A+B is (   )

a) orthogonal b)Unitary c) Skew-symmentric d)Symmentric

6.If A&B are matrices and if AB is defined then the rank of AB is equal to (   )

a)rank of A b)rank of B c)≤min {rank A , rank B} d)≤max {rankA,rankB}

7.(A-λI)is called ………. (   )

a)singular matrix b)non singular matrix c)charecterstic matrix d)proper matrix

8.The eigen values of hermition matrix are …………….. (   )

a)purely b)regid c)real d)imaginary

9.Diagonalise of a matrix D^n =………. (   )

a)PAP b)P^TAP c)P^TA^nP d)$P^{-1}A^nP$

10.The eigen values of skew hermistion matrix are,,, (   )

a)0 b)1 c)-1 d)real

11.if A and B are 3 x 4 matrix ,then the rank of (A+B) IS (   )

a)4 b)≤3 c)≥0 d)none

12.A non zero matrix A is said to be …………. (   )

a)$A^n=0$ b)$A^n= -1$ c)$A^n=1$ d)none

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22. If the eigen values of are 0,0,6 then the rank of quadratic form is ( )

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23.The index and signature of quadratic form $5x^2+2y^2+2z^2+6yz$ are ( )

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25.The matrix $U=\frac{1}{2}\begin{pmatrix} 1+i & -1+i \\ 1+i & 1-i \end{pmatrix}$ is ( )

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23)a 25)b 25) d

G R E E N

1.If A&B are two invertible square matrix ,then the eigen values of AB and BA are ( )

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4. X+Y+W=0; Y+Z=0; X+Y+Z+W=0; X+Y+2Z=0 rank of the matrix is ( )

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5. A square matrix A of order n x n is sometimes called as a ( )

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a) $A^\Theta \pm B^\Theta$ b) $A^\Theta + B^\Theta$ c) $A^\Theta - B^\Theta$ d) $A^\Theta * B^\Theta$

11.Rank of aunit matrix of order 4 is …. ( )

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12.The matrix A= $\begin{pmatrix} a+ic & -b+id \\ b+id & a-ic \end{pmatrix}$ is unitary if and only if $a^2+b^2+c^2+d^2$= ( )

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17.if the eigen values of A or 1,3+$\sqrt{8}$,3-$\sqrt{8}$ then the index and signature of the quadratic form X^TAX are ( )

a) 1,2 b) 3, 1 c) 3,2 d) 3,3

18.If A = $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ find A^{50} ( )

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20.$3x^2 +5xy-2y^2$ is a ____________ in two varabels X & Y ( )

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a)2,2 b)-2,2 c)-1,4 d)1,4

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a)0 only b)0and 1 only c)0 and -1 only d)-1 and 1 only

25.To find the inverse of the matrix using columns operation only we should proceed as follows A =….. ( )

a)AI_m b)AI_n c)AI d)none

KEY:

1)a 2)c 3) d 4)a 5)d 6) d 7) b 8)d 9)c 10) a 11)d 12)b 13) a 14) c 15) b 16)a 17)d 18)b 19)a 20)b 21)a 22)b 23) d 24)b 25)b

UNIT – II
G R E E N

1. The value of of Rolle's Theorem for $f(x) = \sin x/e^x$ in $(0,\Pi)$ I

a) $\Pi/4$ b) $\Pi/2$ c) Π d) None

Ans-a

2. The value of Legrange's mean value theorem for the functions f(x)=sinx g(x)= cosx in [a,b] is

a) a+b b) a+b/2 c) a-b/2 d) None

Ans-b

3. If f(x) is continuous in [a,b] , $f'(x)$ exists for every value of x in (a,b), f(a)=f(b), then there exists atleast one value c of x in (a,b) such that $f'(c)=$

a) 0 b) a+b c) c d) None

Ans-c

4. Taylor's series expansion of f(x) about x=a is]

a) $f(a)-f'(a)(x-a)+f''(a)(x-a)^2/2!+\ldots.+(-1)^n f^n(a)(x-a)^n/n!$

b) $f(a)+(x-a)f'(a)+f''(a)(x-a)^n/2!+f'''(a)(x-3)^3/3!+\ldots\ldots$

c) $f(a)+xf'(a)+x^2f''(a)+\ldots+x^nf^n(a)$

d) None

Ans-b

5. The value of c of rolle's theorem in (-1,1) for $f(x)=x^3-x$ is

a) 0 b) $\pm1/\sqrt{3}$ c) $\pm1/\sqrt{2}$ d) None

Ans-b

6. Is Rolle's theorem applicable to f(x)=|x| in [-1,1]

a) Not applicable(not differential at 0) b) Not applicable(not differential at - c) applicable d) None

Ans-a

7. The value of c in Rolle's mean value theorem for $f(x)=\sin x/e^x$ in $(0,\Pi)$ is

a) Π b) $\Pi/4$ c) $\Pi/3$ d) None

Ans-b

8. The value of c in Lagrange's mean value theorem for $f(x)=e^x$ in[0,1] is

a) loge b) log(e-1) c) 0 d) None

Ans-b

9. Laplace equation in two dimensions is

a) $u_{xx}+u_{yy}=0$ b) $u_x^2+u_y^2=0$ c) $u_{xy}+u_{xx}=0$ d) None

Ans-a

10. The minimum value of $x^2+y^2+z^2$ given that x+y+z=3a is

a)3a b)$1/3a^2$ c)$3a^2$ d)None

Ans-c

11. The function f(x,y) has a maximum value for

a)$ln-m^2>0,l<0$ b)$ln-m^2=0$ c)$ln-m^2<0,l<0$ d)None

Ans-a

12. If u(1-v)=x,uv=y then J[u,v/x,y].J[x,y/u,v]=

a)0 b)1 c)xy d)None

Ans-a

13. If $u=x^3y^2$,we have $x^2-xy+y^2=a^2$,then dy/dx=

a)$x^2y(4x^2+xy-6y^2)/x-2y$ b)0 c)1 d)None

Ans-a

14. If u=x+y/1-xy,v=$\tan^{-1}x+\tan^{-1}y$,then J[u,v/x,y].J[x,y/u,v]

a)0 b)1 c)xy d)None

Ans-a

15. logx-logy is a homogenous function of drgree

a)0 b)1 c)2 d)None

Ans-a

16. If u=J[u,v/x,y] ,then J[x,y/u,v]=

a)u b)1/u c)1 d)None

Ans-b

17. If u=x cosy ,v=y sinx,then $\partial(u,v)/\partial(x,y)$ is

a)cosy sinx+xy siny cosx b)cosy sinx-xy sinx siny c)cosy cosx+siny sinx d)None

Ans-b

18. If x=rcos ,y=rsin ,then J[x,y/r,θ]=

a)r b)tanθ c)0 d)None

Ans-a

19. The stationary points of $x^3y^2(1-x-y)$ are

a)(0,1) b)(-1,-1) c)(1,1) d)None

Ans-a

20. If λ is the Lagrangian multiplier in maximizing 8xyz when $x^2a^2+y^2b^2+z^2c^2=1$ then =

a)b^2y b)$-a^2xyz$ c)2^{2x} d)None

Ans-c

21. The value of x so that $f(b)-f(a)/b-a=f'(x)$ where a<x<b given $f(x)=1/x^2$,a=1,b=4 Is

a)1/2 b)9/4 c)1/4 Ans-b d)None

22. If $u=\sin^{-1}[x+y/\sqrt{x}+\sqrt{y}]$ and $x\partial u/\partial x+\partial u/\partial y=m\tan u$ the m=

a)1/2 b)-1/2 c)1 d)None

Ans-a

23. If u=sin(x+y), then $\partial u/\partial y=$

a)sinx b)cos(x+y) c)tan(x+y) d)None

Ans-b

24. If f(x,y)=c,ehere c is constant then $\partial y/\partial x=$

a)$-f_x/f_y$ b)0 c)f_x/f_y d)None

Ans-a

25. The degree of homogenous functionz=$\sqrt{x}+\sqrt{y}/x+y$=

a)1/2 b)-1/2 c)0 d)None

Ans –b

Y E L L O W

1. logx-logy is a homogenous function of degree
   a)1 b)0 c)1/2 d)None

Ans-b

2. If $u=x^y$ then $\partial^2u/\partial x\partial y=$
   a)$yx^{y-1}(1+y\log x)$ b)0 c)yx^{y-1} d)None

Ans-a

3. Two functions u and v are said to be functionally dependent if $\partial(u,v)/\partial(x,y)=$
   a)0 b)1 c)not defined d)None

Ans-a

4. If f is a function of u,v,w and u,v,w are the functions of x,y,z then $\partial f/\partial y$ is
   a)0 b)$\partial f/\partial u.\partial u/\partial z+\partial f/\partial v.\partial v/z+\partial f/\partial w.\partial w/\partial z$ c)1 d)None

Ans-b

5. The stationary points of $x^4+y^4-2x^2+4xy-2y^2$ is
   a)$(\sqrt{2},\sqrt{2})$ b)$(\sqrt{2},0)$ c)$(\sqrt{2},-\sqrt{2})$ d)None

Ans-a

6. J[u,v/x,y]=
   a)0 b)1 c)1/2 d)None

Ans-b

7. The value of c of rolle's theorem for $f(x)=\sin x/e^x$ in $(0,\Pi)$ is
   a)Π b)$\Pi/2$ c)$\Pi/4$ d)None

Ans-b

8. In Taylor's theorem Cauchy's form of remainder is
   a)$h^{n-1}f^{n-1}(a-\theta h)/n!$ b)$h^nf^n(a+\theta h)$ c)$h^n(1-\theta)^{n-1}f^n(\theta h+a)/(n-1)!$ d)None

Ans-c

9. Legrangre's mean value theorem for f(x)=secx in$(0,2\Pi)$ is
   a)applicable b)not applicable due to discontinuity c)applicable and $c=\Pi/2$
d)None

Ans=b

10. Is the Rolle's theorem applicable for $f(x)=x^2$ in [1,2]
   a)not applicable[$f(1)\neq f(2)$] b)applicable c)not applicable[f(1)=f(2)]
d)None

Ans=a

11. Using which mean value theorem, we can calculate approximately the value of $(65)^{1/6}$ in an easier way
   a)Cauchy's b)Legrange's c)Rolle's d)None

Ans-b

12. If z is a homogenous function of degree n,in x,,y,z=f(u), then $xu_x+yu_y=$
   a)nf(u)   b)0   c)$nf'(u)$ d)None

Ans-a

13. If u=sin(ax+by+cz),then $\partial u/\partial x=$
   a)acos(ax+by+cz)   b)asin(ax+by+cz)   c)bcos(ax+by+cz)   d)None

Ans-a

14. If $u=\sin(xy^2)$ we have $x=\log t, y=e^t$ then du/dt=
   a)$y^2[1/t+2x]\cos xy^2$ b)0   c)1   d)None

Ans-a

15. If $u=\log(x^2+y^2)$ then $\partial u/\partial x=$
   a)2y/x+y   b)2y/x-y   c)$2x/x^2+y^2$ d)None

Ans-c

16. If z is a homogenous function of degree n then $x^2+z_{xx}+2xyz_{xy}+y^2z_{yy}=$
   a)nz   b)0   c)n(n-1)z   d)None

Ans-c

17. If u= $\Phi(y+ax)+\Psi(y-ax)$ then $\partial^2u/\partial x^2-a^2\partial^2u/\partial x^2=$
   a)0   b)1   c)2   d)None

Ans-a

18. The value of legrange's mean value theorem for the function $f(x)=x^2$ in[1,5] is
   a)3   b)0   c)1   d)None

Ans-a

19. The value of Cauchy's mean value theorem for the functions $f(x)=x^2,g(x)=x^3$ in The interval [1,2] is
   a)14/9   b)14/5   c)17/9   d)None

Ans-a

20. The value of Rolle's mean value theorem in (-1,1) for $f(x)=x^3-x$ is
   a)0   b)$\pm1/\sqrt{3}$ c)1/2   d)None

Ans-b

21. If $u=x^y$ then $\partial u/\partial x=$
   a)yx^{y-1} b) 0   c)$x^{y-1}y$ d)None

Ans-a

22. If u=x/y and v=x+y/x-y then J[u,v/x,y]=
   a)0   b)1   c)1/2   d)None

Ans-a

23. If $u=\tan^{-1}[y/x]$,then $x\,\partial u/\partial x+y\partial u/\partial y=$
   a)0   b)sin 2u   c)cos 2u   d)None

Ans-a

24. If f(x) is continuous in [a,b],$f^{'}(x)$ exists for every value of x in (a,b),f(a)=f(b),t6hen there exists at least one value c of x in (a,b) such that $f^{'}(c)$=

a)0 b)a+b c)c d)None

Ans-a

25. The value of c of rolle's mean va;ue theorem in [1/2,2] for f(x)=x^2+1/x^2 is

a)3/4 b)5/4 c)1 d)None

Ans-c

R E D

1. The stationary points of $x^3y^2(1-x-y)$ are
   a)(0,1) b)(-1,-1) c)(1,1) d)None
Ans-a
2. The value of Cauchy's mean value theorem for the functions $f(x)=e^x$ and $g(x)=e^{-x}$ in [a,b] is
   a)0 b)a+b/2 c)a-b/2 d)None
Ans-b
3. If u=3x+y v=x-2y then $\partial(u,v)/\partial(x,y)$ is
   a)-6 b)7 c)-7 d)None
Ans-c
4. If $u=xy^2\Phi[x/y]$ then $x\partial u/\partial x+y\partial u/\partial y=$
   a)0 b)1 c)u d)None
Ans-c
5. If $x=r\cos\theta$ $y=r\sin\theta$ then $\partial\theta/\partial r,\partial y/\partial\theta$ are
   a)$\cos\theta$,r $\cos\theta$ b)$\cos\theta,\sin\theta$ c)$\cos\theta,\sec\theta$ d)None
Ans-a
6. If $u=\tan^{-1}[x^3+y^3/x-y]$ then $x\partial u/\partial x+y\partial u/\partial y=$
   a)sin 2u b)cos 2u c)0 d)None
Ans-a
7. Is rolle's theorem applicable to the function $f(x)=1/x^2$ in [-1,1]
   a)not applicable b)applicable c)not applicable(at -1) d)None
Ans-a
8. $\partial(u,v)/\partial(x,y)*\partial(x,y)/\partial(u,v)=$
   a)1 b)0 c)1/2 d)None
Ans-a
9. If $u=e^x$ $v=e^x\cos y$ then $\partial(u,v)/\partial(x,y)$
   a)$-e^x$ b)e^x c)$-e^{-x}$ d)None
Ans-a
10. If u=f(y/x) then $\partial u/\partial y=$
   a)xy logx b)xy c)logx d)None
Ans-a
11. The value of legrange's mean value theorem for $f(x)=x^2-3x+2$ in [-2,3] is
   a)1/2 b)1 c)0 d)None
Ans-a

12. If $x=r\cos\theta$ $y=r\sin\theta$ $\partial r/\partial x=$,$\partial r/\partial y=$

a)x/r,tanθ b)x/r,y/r c)tanθ,sinθ d)None

Ans-b

13. If u=J[u,v/x,y] then J[x,y/u,v]

a)u b)1/u c)1 d)None

Ans-a

14. If $u=x^2y, v=xy^2$ then $\partial(u,v)/\partial(x,y)$ is

a)$5x^2y^2$ b)$4x^2y^2$ c)$2x^2y^2$ d)None

Ans-1

15. If $u=e^{x/y}$ then$xu_x+yu_y=$

a)0 b)1 c)2 d)None

Ans-a

16. Are $u=x\sqrt{1-x^2}$,v=2x functionally dependent?If so, what us J[u,v/x,y]?

a)yes,1 b)yes,0 c)no,0 d)None

Ans-b

17. The value of c of cauchy's mean value theorem for the functions $f(x)=\sqrt{x}$, $g(x)=1/\sqrt{x}$ in [a,b] is

a)a+b/2 b)a-b/2 c)a+b d)None

Ans-a

18. If $J=\partial(u,v)/\partial(x,y)$ $J'=\partial(x,y)/\partial(u,v)$ then $JJ'=$

a)1 b)0 c)3 d)None

Ans-a

19. If f(x,y)=xy+(x-y),the stationary points are

a)(0,0) b)(1,-1) c)(1,1) d)None

Ans-b

20. If $u=x^2-2y, v=x+y$ then $\partial(u,v)/\partial(x,y)=$

a)$(x+1)^2$ b)2(x+1) c)3(x+1) d)None

Ans-b

21. If $u=\tan^{-1}y/x$ then $\partial u/\partial x$ at (0,-1) is

a)1 b)0 c)2 d)None

Ans-a

22. If u=x siny+y sinx then $y_{xy}-u_{yx}=$

a)1 b)0 c)2 d)None

Ans-b

23. If $u=\log(x^3+y^3+z^3-3xyz)$ then $x\partial u/\partial x+y\partial u/\partial y=$

a)4u b)3u c)u d)None

Ans-c

24. If $f=x^2+y^2$ then $\partial^2 f/\partial x\partial y=$

a)1 b)0 c)-1 d)None

Ans-c

25. Is rolle's theorem applicable for the function f(x)=tanx in[0,Π]

a) Not applicable (discontinuous at x=Π/2) b) applicable c) not applicable (at Π) d)None

Ans-a

UNIT-IV

Partial Differential Equations

INTRODUCTION

D.E.: An Equation involving a dependent variable and differential coefficient of the dependent variable with respect to one a more than one independent variables is called a D.E.

Partial Differential Equations

A. D.E. in which the diffentials involved are with reference to two or more than two independent variables is called partial differential equation.

Ex : $x\frac{\partial z}{\partial y}+4y\frac{\partial z}{\partial x}=2z+3xy$

Linear & Non linear P.D.E

If the partial derivatives as well as the dependent variable occur in first degree only and separately, Such a P.D.E is said to the linear P.D.E..

Otherwise it is a non –linear P.D.E.

Homogeneous & Non Homogeneous P.D.E

A P.D.E is said to the Homogeneous it each term of the equation entairs either the dependent variable as one of its derivations

Otherwise it is said to be Non - Homogeneous

Notations

If μ = f(x,y), we employ the following rotations

$$\frac{\partial\mu}{\partial x} = p, \frac{\partial\mu}{\partial y} = q, \frac{\partial^2\mu}{\partial x\partial y} = s, \frac{\partial^2\mu}{y^2} = t$$

Formation & partial Differential Equations.

Partial Differential equations can be formed by two methods

1. By the elimination of arbitrary constants
2. By the elimination of arbitrary functions

I. Method of Elimination of arbitrary contants

If the number of arbitrary constants to be eliminated is equal to the number of independent variables, we get a partial differential equation of first order.

If the number of arbitrary constants to be eliminated is grater than the number of independent variables we get a partial differential equation of higher order.

1. From the partial differential equation by eliminating the arbitrary constants from.

Z = ax+by+ab

Let Z = ax +by +ab ------------1

Diff (1). Partially w.r.t x and y

p = $\frac{\vartheta z}{\vartheta x}$ and q= $\frac{\vartheta z}{\vartheta y}$

=a = b

By eq 1 we have

Z = px+qy+pq

Which is the required D.E.

Linear partial Differential Equations of First Order

Lagrange's Linear Equation:

A linear partial differential equation of the first order, commonly known as Lagrange's linear equation is of the form

$$Pp + Qq = R$$

Where P,Q and R are functions of x,y,z

And p = $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

Method of solving

To solve the equation Pp+Qq = R

(i). form the auxiliary equations

$$dx/P = dy/Q = dz/R$$

(ii). Solve the auxiliary equations obtaining two independent solutions u =a and v =b

(iii). Then the solution is $\phi(u,v)=0$ or u = F(v) or v=F(u).

Solution of the subsidiary equations

The subsidiary equations are

dx/P =dy/Q= dz/R

I Method of Grouping:

The equations are dx/P =dy/Q= dz/R by taking first two members dx/P =dy/Q and then

Integrating we get an equation, say u=a which gives one equation of the solution similarly, taking any other two members, and then integrating, we get another equation, say v= b which gives another equation of the solution.

II Method of Multiplies

If *l*,m,n are functions of x,y,z are constants, then

$$dx/P = dy/Q = dz/R = \frac{ldx + mdy + ndz}{lP + mQ + nR}$$

now if the multiplies *l*,m,n are chosen in such a way that *l*P+mQ+nR = 0 then *l*dx+mdy+ndz =0. Integrating we get u =a. This gives one equation of the solution.

Similarly, taking multiplies l^1, m^1, n^1 in such a way that $l^1P+m^1Q+n^1R = 0$ then $l^1dx+m^1dy+n^1dz = 0$. Integrating v =b. this gives another equation of solution.

The two solution u=a, v=b so obtained form the complete solution.

Note: We may get one solution u=a from the method of grouping and another solution *v*=b from the method of multiplies.

<u>Problems</u>

1). Solve x(y-z)p+y(z-x)q=z(x-y).

Sol:- The Auxiliary equations are

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

Taking 1,1,1 as multiplies

Each fraction = $\frac{dx + dy + dz}{x(y-z) + y(z-x) + z(x-y)} = \frac{dx + dy + dz}{0}$

This gives dx+dy+dz = 0

$\Rightarrow$ x+y+z =a (u=a)

Again taking 1/x, 1/y, 1/z as multiplies

Each fraction = $\dfrac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{x(y-z)+y(z-x)+z(x-y)}$

$= \dfrac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{0}$

This gives $\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz$ =0

$\Rightarrow$log x+log y+log z = log b

$\Rightarrow$ xyz =b (v=b)

The general integral is

ϕ(x+y+z, xyz) =0.

2). Solve $y^2zp+x^2zq = xy^2$

Sol: Auxiliary equations are

$$\frac{dx}{y^2z}=\frac{dy}{x^2z}=\frac{dz}{xy^2}$$

From $\frac{dx}{y^2z}=\frac{dy}{x^2z}$

$\Rightarrow x^2dx = y^2dy$

$\Rightarrow x^3/3 = y^3/3+a$

$\Rightarrow x^3-y^3 = a^1$ where $a^1= 3a$ (u=a)

Again $\frac{dx}{y^2z}=\frac{dz}{xy^2}\Rightarrow$ xdx = zdz

$\Rightarrow x^2/2 = z^2/2+b$

$\Rightarrow x^2-z^2 = b^1$ where $b^1 = 2b$ ($v=b^1$)

The general integral is

$\phi(x^3-y^3, x^2-z^2) = 0.$

3). Solve $(x^2-y^2-z^2)p+2xyq = 2xz$

Sol: Auxiliary equations are

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

from $\frac{dy}{2xy} = \frac{dz}{2xz}$

$$\frac{dy}{y} = \frac{dz}{z}$$

$\Rightarrow \log y = \log z + \log a$

$\Rightarrow y/z = a$ $(u=a)$

Again using x,y,z as the multieplies

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} = \frac{xdx+ydy+zdz}{x(x^2+y^2+z^2)}$$

Now $\frac{xdx+ydy+zdz}{x(x^2+y^2+z^2)} = dz/2xz$

$$\Rightarrow \frac{2(xdx + ydy + zdz)}{x^2 + y^2 + z^2} = dz/z$$

Integrating

$Log(x^2+y^2+z^2) = \log z + \log b$

$$\frac{x^2 + y^2 + z^2}{z} = b \qquad (v = b)$$

The general integral is $\phi(y/z, \frac{x^2 + y^2 + z^2}{z})=0$

Non- Linear Equations of First order

A partial differential equation of first order but of degree more than one is called a non-linear partial differential equation.

Standard Form I:

Equations involving only p,q and not x,y,z.

i.e $f(p,q) = 0$ --------(1)

an integral of (1) is given by

$z=ax+by+c$-----------(2)

where a and b are connected by the relation

$f(a,b)=0$----(3)

since from (2) $p = \frac{\partial z}{\partial x} = a$ and $\frac{\partial z}{\partial y} = b$

which when substituted in (3) yields (1)

i.e (2) satisfies the given equation

now solving (3) for b, let $b = F(a)$. putting this value of b in (2), the complete integral is given by

$z = ax + y\, F(a) + c$ -----------(4)

The singular integral is obtained by eliminating a and c between the complete integral (4) and the equations obtained by differentiating (4) w.r.t 'a' and c.

Standard Form IV:

$Z = px + qy + f(a,b)$

Clairaut's Type:

Equations of this type have form

$Z = px + qy + f(p,q)$--------(1)

We can easily verify that a solution 1 is

$Z = ax + by + f(a,b)$-------------(2)

Where a, b are arbitrary constants, therefore it is the complete integral.

Partially differentiating (2) w.r.t a and b in turn and equating to zero the results derived, we have the equations.

$0 = x + \partial f/\partial a$--------(3)

And $0 = y + \partial f/\partial b$----------------(4)

Eliminating a and b from the equations (2), (3) and (4) we get singular solution.

To obtain the general integral, we put $b = \phi(a)$ in (2), where ϕ is an arbitrary function.

Then $z = ax + y\,\phi(a) + f[a, \phi(a)]$ --------------(5)

Partially differentiating (5) w.r.t a and equating it to zero we get

$0 = x + y\,\phi^1(a) + f^1(a)$ --------------------(6)

The elimination of a between the equations (5) and (6) is the general integral.

Standard Form II:

Equation does not involve x and y

i.e $f(z,p,q) = 0$ ------------------(1)

we take $q= ap$ -------------(2)

where a is an orbitary constant.

Solve (1) and (2) for p in terms of z say, we obtain

$P=\phi(z)$----------------(3)

$dz = pdx+qdy$

$= pdx+a pdy$

$=p(ax+ady)$

$dx+ady = dz/\phi(z)$ ------------------(4)

integrating (4),

$$x+ay = \int \frac{dz}{\phi(z)} + b \text{ ----------(5)}$$

which is the complete integral of (1) working rule of solve $f(p,q,z)= 0$;

1. Let us assume $u = x+ay$ and using $p= dz/du$ and $q = adz/du$ in the given equation

   $f(z,p,q) = 0$ and which transform into $f(z,dz/du, adz/du) = 0$.

2. Solve the resulting ordinary differential equation

   $f(z,dz/du, adz/du) =0$

3. Substituting x+ay in place of u.

STANDARD FORM III. VARIABLES SEPARABLE

Equation of the form $f_1 (x,p) = f_2 (y,q)$ i.e. equations not involving z and the terms containing x and p can be separated from those containing y and q.

As a trail solution, we assume each side equal to an arbitrary constant a, solve for p and q from the resulting equation.

$f_1(x,p) = a$ and $f_2(x,p)=a$

Solving for p and q, we obtain

$P = F_1(x,a)$ and $q= F_2(y,a)$

Since z is a function of x and y, we have

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = pdx = qdy$$

$$\therefore dz = F_1(x,a)dx + F_2(y,a)dy + b$$

Integrating z = $\int F_1(x,a)dx + \int F_2(y,a)dy + b$

Which is the required complete solution containing two arbitrary constants a and b.

Example : Solve $p - q = x^2+y^2$

Solution: Seperating p and x from q and y, the given equation can be written as $p-x^2=q+y^2=a$, (say)

$\therefore p - x^2 = a$ gives $p = a+x^2$ and $q=y^2=a$ gives $q=a-y^2$

Putting the values of p and q and dz = pdx + qdy, we get

$dz= (a=x^2)\ dx+ (a-y^2)\ dy$

Integrating z = ax+ $\frac{x^3}{3} + ay - \frac{y^3}{3} + b = \frac{1}{3}(x^2 - y^3) + a(x+y) + b$

Which is the desired solution.

Example : Solve $p^2+q^2 = x^2+y^2$

Solution: Given equation can be written as

$p^2- x^2= y^2- q^2= a$, say

$$\therefore p^2 - x^2 = a \Rightarrow p = \sqrt{x^2 + a}$$

and $y^2 - q^2 = a \Rightarrow p = \sqrt{y^2 - a}$

Substituting these values of p and q in dz= pdx + qdy, we get

$$dz = \sqrt{x^2 + a}dx + \sqrt{y^2 - a}dy$$

Integrating,we get

$$\int dz = \int \sqrt{x^2 + \left(\sqrt{a}\right)^2}dx + \int \sqrt{y^2 - \left(\sqrt{a}\right)^2}dy$$

$$\Rightarrow z = \frac{x}{2}\sqrt{x^2 + a} + \frac{a}{2}\sinh^{-1}\frac{x}{\sqrt{a}} + \frac{y}{2}\sqrt{y^2 - a} - \frac{a}{2}\cosh^{-1}\frac{x}{\sqrt{a}} + c$$

$$= \frac{1}{2}\left(x\sqrt{x^2 + a} + y\sqrt{y^2 - a}\right) + \frac{a}{2}\left(\sinh^{-1}\frac{x}{\sqrt{a}} - \cosh^{-1}\frac{x}{\sqrt{a}}\right) + c$$

Which is the required solution

ONE DIMENSIONAL WAVE EQUATION

Let OA be a stretched string of length l with fixed ends O and A. Let us take x-axis along OA and y-axis along OB perpendicular to OA, with O as origin. Let us assume that the tension T in the string is constant and large when compares with the string so that the effects of gravity are negligeable. Let us pluck the string in the BOA plane and allow it to vibrate. Let p be any point of the string at time t. Let there be no external forces acting on the string. Let each point of the string make small vibrations at right angles to OA in the plane of BOA. Draw pp^1 perpendicular to OA. Let $op^1 = x$ and $pp^1 = y$. Then y is a function of x and t. Under the assumptions, using Newton's Second Law of motion, it can be proved that $y(x,t)$ is governed by the equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} ----------- (1)$$
$$i,e., \frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$
$$where\ \ c^2 = T/m$$

With T = tension in the string at any point and m is mass per unit length of the string.

Since the points O and A are not disturbed from their original positions for any time t we get

$$y(0,t) = 0 ------ (2)$$

$$y(1,t) = 0 ------ (3)$$

These are referred to as the end conditions or boundary conditions. Further it is possible that, we describe the initial position of the string as well as the initial velocity at any point of the string at time $t = 0$ through the conditions

$$y(x,O) = f(x), 0 \le x \le l ------ (4)$$
$$\frac{\partial y}{\partial t}(x,O) = g(x), 0 \le x \le l ----- (5)$$

Where $f(x)$ and $g(x)$ are functions such that $f(O) = f(l) = 0$;and $g(O) = g(l) = 0$. Thus to study the subsequent motion of any point of the string we have to solve following :

Determine $y(x,t)$ such that $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} ----- (1)$

Subject to the condition

$$\left.\begin{array}{l} y(O,t) = 0\ \ for\ all\ t ---- (2) \\ y(l,t) = 0\ \ for\ all\ t ---- (3) \end{array}\right\} end\ condtions$$

$$\left.\begin{aligned} y(x,O) &= f(x), O \le x \le 1----(4) \\ \left(\frac{\partial y}{\partial t}\right)_{at\ t=0} &= g(x), O \le x \le 1----(5) \end{aligned}\right\} initial\ condtions$$

The equation (1) is called one dimensional wave equation

Solution of equation (1) to (5)

Consider the equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} ------(1)$

Let us use the method of separation of variables. Here $y = y(x,t)$. Let us take $y = X(x)T(t)$

As solution of (1). Then

$$\frac{\partial y}{\partial x} = X^1(x)T(t); \frac{\partial^2 y}{\partial x^2} = X^{11}(x)T(t);$$
$$\frac{\partial y}{\partial t} = X(x)T^1(t); \frac{\partial^2 y}{\partial t^2} = X(x)T^{11}(t)$$

Using these in (1) we get

$$X^{11}(x)T(t) = \frac{1}{c^2}X(x)T^{11}(t)$$
$$\therefore \frac{X^{11}(x)}{X(x)} = \frac{1}{c^2}\frac{T^{11}(t)}{T(t)}$$

Since the left hand side is function of x and right hand side is a function of t the equality is possible if and only if each side is equal to the same constant (say) λ.

Hence we shall take

$$\frac{X^{11}(x)}{X(x)} = \frac{1}{c^2}\frac{T^{11}(t)}{T(t)} = \lambda$$

Let us take λ to be real. Then three cases are possible $\lambda > 0, \lambda = 0\ or\ \lambda < 0$

Case 1:- let $\lambda > 0$, then $\lambda = p^2\ (p > 0)$

Then $$\frac{X^{11}(x)}{X(x)} = \frac{1}{c^2}\frac{T^{11}(t)}{T(t)} = p^2$$

Hence $X^{11}(x) = p^2 X(x) (i.e.,) X^{11}(x) - p^2 X(x) = 0$

$$i.e., \frac{d^2X}{dx^2} - p^2 X = 0 \Rightarrow X(x) = A_1 e^{px} + B_1 e^{-px}$$
$$Also\ T^{11}(t) - p^2c^2T(t) = 0$$
$$\Rightarrow T(t) = C_1 e^{pct} + D_1 e^{-pct}$$

Hence in this case, a typical solution is like

$$y(x,t) = \left(A_1 e^{px} + B_1 e^{-px}\right)\left(C_1 e^{pct} + D_1 e^{-pct}\right) ------(S.1)$$

Where A_1, B_1, C_1, D_1 are arbitary constants

Case 2:- let $\lambda - 0$ then

$$\frac{X^{11}(x)}{X(x)} = \frac{T^{11}(t)}{C^2T(t)} = 0$$

$$\therefore X^{11}(x) = 0 \Rightarrow X(x) = A_2 + B_2 x$$

$$T^{11}(t) = 0 \Rightarrow T(t) = C_2 + D_2 t$$
$$\therefore y(x,t) = (A_2 + B_2 x)(C_2 + D_2 t) + ------(S.2)$$

Where A_2, B_2, C_2, D_2 are arbitary constants

Case 3:- Let $\lambda < 0$. Then we can write $\lambda = -p^2$ where $p > 0$ then

$$\frac{X^{11}(x)}{X(x)} = \frac{T^{11}(t)}{c^2T(t)} = -p^2$$
$$\therefore X^{11}(x) + p^2 X(x) = 0$$
$$\Rightarrow X(x) = (A_3 \cos px + B_3 \sin px)$$
$$T^{11}(t) + p^2c^2T(t) = 0$$

$$\Rightarrow X(t) = (C_3 \cos pct + D_3 \sin pct)$$

Hence a typical solution in this case is

$$y(x,t) = (A_3 \cos px + B_3 \sin px)(C_3 \cos pct + D_3 \sin pct)$$

Thus the possible solution forms of equation (1) are

$$y(x,t) = (A_1 e^{px} + B_1 e^{-px})(C_1 e^{pct} + D_1 e^{-pct}) ----(S.1)$$
$$y(x,t) = (A_2 + B_2 x)(C_2 + D_2 t) --------(S.2)$$
$$y(x,t) = (A_3 \cos px + B_3 \sin px)(C_3 \cos pct + D_3 \sin pct) ---(S.3)$$

Consider $(S.1)$ (I.e.,)

$$y(x,t) = (Ae^{px} + Be^{-px})(Ce^{pct} + De^{-pct})$$

Using conditions (2) $(viz)\, y(0,t) = 0 \ for \ all \ t$

$$(A+B)(Ce^{pct} + De^{-pct}) = 0 \ for \ all \ t$$

$$\therefore A + B = 0$$

Using condition (3), $y(l,t) = 0 \ for \ all \ t$

$$\therefore (Ae^{pl} + Be^{-pl})(Ce^{pct} + De^{pct}) = 0 \ for \ all \ t$$
$$\therefore Ae^{pl} + Be^{-pl} = 0$$

Solving $A + B = 0$

And $Ae^{pl} + Be^{-pl} = 0$

We get $A = B = 0$

Thus $y(x,t) = 0$

This implies that there is no displacement for any x and for any t. this is impossible. Thus $(S.1)$ is not an appropriate solution

Consider $(S.2)$:

$$y(x,t) = (A + Bx)(C + Dt)$$

Using (2), $y(0,t) = 0 \ for \ all \ t$

Hence $A(C + Dt) = 0 \Rightarrow A = 0$

Using (3), $y(l,t) = 0 \ for \ all \ t$

$$\therefore (A + Bl)(C + Dt) = 0 \ for \ all \ t$$

$$\therefore Bl(C + Dt) = 0 \forall t \quad \text{since A = 0}$$

Here $l \neq 0; C + Dt \neq 0 \forall t$ Hence B = 0

Thus here again $y(x,t) \equiv 0 \forall x \ and \ t$

Thus as before, this solution also is not valid

Hence $(S.2)$ is also not appropriate for the present problem

Consider $(S.3)$

$y(x,t) = (A\cos px + B\sin px)(C\cos pct + D\sin pct)$ (using condition 2)

$$y(x,t) = 0 \forall t$$
$$\Rightarrow A(C\cos pct + D\sin pct) = 0$$
$$\Rightarrow A = 0$$

Using condition 3

$y(l,t)=0 \forall t$

$B \sin pl\left(C \cos pct + D \sin pct\right)=0$

$if\;\; B=0, y(x,t)=0$ and this is invalid

Hence $\sin pl = 0$

$\therefore pl = n\pi$ where n = 1, 2, 3…………

Thus $p=\frac{n\pi}{l}(n=1,2,3,.......)$

Thus a typical solution of (1) satisfying conditions (2) & (3) is

$$y(x,t)=\sin\frac{n\pi x}{l}\left[C_n \cos\frac{n\pi ct}{l}+D_n \sin\frac{n\pi ct}{l}\right]$$
$for\;\; n=1,2,3......$

Since different solutions correspond to different positive integer n.

<u>An Important observation here :</u>

If $\left[y_n(x,t)\right]_{n=1}^{\infty}$ are functions satisfying (1) as well as conditions (2) and (3). As the equation (1) is linear. The most general solution of (1) here is $y(x,t)=\sum_{n=1}^{\infty} y_n(x,t)$

Thus the most general solution of (1) satisfying (2) & (3) is

$$y(x,t)=\sum_{n=1}^{\infty}\left[C_n \cos\frac{n\pi ct}{l}+D_n \sin\frac{n\pi ct}{l}\right]\sin\frac{n\pi x}{l} \rightarrow (6)$$

Where $C_n \; and \; D_n$ are constants to be determined using (3) and (4)

Let us use condition 4: $y(x,0)=f(x), 0\le x\le l$

Thus putting t = 0 in (6)

$$\sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{l} = f(x), 0\le x\le t$$

Hence $C_n = 2/l\int_0^l f(x)\sin\frac{n\pi x}{l}dx$ n = 1, 2, ……

Thus $C_n's$ are all determined

Let us consider condition (5):

$$\left(\frac{\partial y}{\partial t}\right)_{at\, t=0} = g(x)\,\forall 0\le x\le t$$

$$\frac{\partial y}{\partial t} = \sum_{n=1}^{\infty}\left\{\left(-C_n\sin\frac{n\pi ct}{l}\left(\frac{n\pi c}{l}\right)+D_n\cos\frac{n\pi ct}{l}\left(\frac{n\pi c}{l}\right)\right)\sin\frac{n\pi x}{l}\right\}$$

$$\left.\frac{\partial y}{\partial t}\right|_{at\, t=0} = g(x)$$

$$\Rightarrow \sum_{n=1}^{\infty}\left(D_n\frac{n\pi c}{l}\right)\sin\frac{n\pi x}{l} = g(x), 0\le x\le l$$

Hence $D_n = \frac{2}{n\pi c}\int_0^t g(x)\sin\frac{n\pi x}{l}dx \quad for(n=1,2.....)$

Thus D_n are all determined

Hence the displacement $y(x,t)$ at any point x and at any subsequent time t is given by

$$y(x,t)=\sum\left(C_n\cos\frac{n\pi ct}{l}+D_n\sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi x}{l}\rightarrow(6)$$

Where $C_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx\rightarrow(7)$

$$D_n = \frac{2}{n\pi c}\int_0^l g(x)\sin\frac{n\pi x}{l}dx\rightarrow(8)$$

TWO DIMENSIONAL WAVE EQUATION:-

Two dimensional wave equation is given by

$$\frac{\partial^2 u}{\partial t^2} = C^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - - - - - (1)$$

Where $C^2 = T/P$, for the unknown displacement $u(x, y, t)$ of a point (x, y) of the vibrating membrane from rest $(\mu = 0)$ at time t.s

The boundary conditions (membrane fixed along the boundary in the xy- plane for all times $t \geq 0$, are $u = 0$ on the boundary ----(2)

And the initial conditions are

$$u(x, y, 0) = f(x, y) : u_t(x, y, o) = g(x, y) - - - - (3)$$

$$where\ u_t = \frac{\partial u}{\partial t}$$

Now we have to find a solution of the partial differential equation (1) satisfying the conditions (2) and (3) . we shall do this in 3 steps, as follows:

Working rule to solve two – dimensional wave equation :-

Step1: By the "method of separating variables" setting $u(x,y,t)=F(x,y),G(t)$ and later $F(x,y)=H(x)Q(y)$ we obtain from (1) an ordinary differential equation for G and one partial differential equation for F, two ordinary differential equations for H & Q.

Step 2: We determine solutions of these equations that satisfy the boundary conditions (2). Step(2) to obtain a solution of (1) satisfying both (2) and (3). That is the solution of the regular membrane as follows.

The double Fourier series for $f(x,y)=\left[u(x,y,0)\right]$ is given by

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}u_{mn}(x,y,t)$$
$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[B_{mn}\cos\lambda_{mn}t+B^{*}{}_{mn}\sin\lambda_{mn}t\right]\sin\frac{n\pi x}{a}\sin\frac{n\pi y}{b}$$

Hence B_{mn} and $B^{*}{}_{mn}$ are called Fourier co-efficients of $f(x,y)$ and are given by

$$B_{mn}=\frac{4}{ab}\int_{0}^{b}\int_{0}^{a}f(x,y)\sin\frac{n\pi x}{a}\sin\frac{n\pi y}{b}dxdy, m=1,2.......;n=1,2.......$$
$$and\ B^{*}{}_{mn}=\frac{4}{ab\lambda_{mn}}\int_{0}^{b}\int_{0}^{a}g(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy, m=1,2.......;n=1,2.......$$

1. **Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2}=C^2\frac{\partial^2 u}{\partial x^2}$ corresponding to the triangular initial deflection**

$$f(x)=\frac{2kx}{l}\ where\ 0<x<l/2$$
$$=\frac{2k}{l}(l-x)\ where\ l/2<x<l$$

and initial velocity is equal to 0.

Ans. To find $u(x,t)$ we have to solve

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\rightarrow(1)$$

Where

$$u(0,t)=0\forall t\rightarrow(2)$$
$$u(l,t)=0\forall t\rightarrow(3)$$
$$u(x,0)=f(x)(0\le x\le l)\rightarrow(4)$$

$$\left(\frac{\partial u}{\partial t}\right)_{at\ t=0}=g(x)=0(0\le x\le l)\rightarrow(5)$$

Equation (1) can be in the form

$$u(x,t)=T(t)X(x)$$

The three solutions of (1) are

$$u(x,t)=\left(A_1e^{px}+B_1e^{-px}\right)\left(C_1e^{pct}+D_1e^{-pct}\right)----(S.1)$$
$$u(x,t)=(A_2+B_2x)(C_2+D_2t)----(S.2)$$
$$u(x,t)=(A_3\cos px+B_3\sin px)(C_3\cos pct+D_1\sin pct)----(S.3)$$

The appropriate solution is S.3

Hence $u(x,t)=(A\cos px+B\sin px)(C\cos pct+D\sin pct)$

Using (2) & (3)

$$A=0;P=\frac{n\pi}{l}\ where\ n=1,2,3......$$

$\therefore$ The most general solution of (1) satisfying (2) & (3) is

$$u(x,t)=\sum_{n=1}^{\infty}\left(C_n\cos\frac{n\pi ct}{l}+D_n\sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi x}{l}\rightarrow(6)$$

Using (4)

$$u(x,0)=f(x)$$
$$\therefore f(x)=\sum C_n\sin\frac{n\pi x}{l}\forall x\in[0,l]\rightarrow(7)$$

Now we can expand the given function $f(x)$ in a half range fourier sine series for $0 < x < l$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \quad where\ b_n = \frac{2}{l}\int_0^l f(x).\sin\frac{n\pi x}{l}\,dx \to (8)$$

Comparing (7) & (8) we get $c_n = b_n$

$$\therefore c_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\left[\int_0^{l/2} \frac{2k}{l} x\sin\frac{n\pi x}{l}\,dx + \int_{l/2}^{l} \frac{2k}{l}(l-x)\sin\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{4k}{l^2}\left[\left\{x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - 1\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right\}_0^{l/2} + \left\{(l-x)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (-1)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)_{l/2}^{l}\right\}\right]$$

$$= \frac{4k}{l^2}\left[l/2.\frac{1}{n\pi} - \cos\frac{n\pi}{2} + \frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

The required solution of (1) is of the form

$$u(x,t) = (c_1\cos px + c_2\sin px) + (c_3\cos pat + c_4\sin pat) \to (6)$$

Using (2) & (3), we have

$$c_1 = 0 \ \ and\ \ p = \frac{n\pi}{l} \ \ where\ n = 1,2,3.....$$

$\therefore$ General solution of (1) satisfying (2) & (3) is

$$u(x,t) = c_2\sin\frac{n\pi x}{l}\left(c_3\cos\frac{n\pi at}{l} + c_4\sin\frac{n\pi at}{l}\right) \to (7)$$

Now using condition (4) $u(x,0) = 0$ we get

$$u(x,0)=0=c_2\sin\frac{n\pi x}{l}(c_3+0)$$

$$\Rightarrow c_2c_3\sin\frac{n\pi x}{l}=0\Rightarrow c_3=0\because(c_2\neq 0)\rightarrow(8)$$

$$from\ (7)\&(8)$$

$$u(x,t)=c_2\sin\frac{n\pi x}{l}\left(0+c_4\sin\frac{n\pi at}{l}\right)$$

$$=c_4\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}\quad where\, c_n=c_2c_4$$

The most general solution of (1) is

$$u(x,t)=\sum_{n=1}^{\infty}c_n\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}----(9)$$

$$\frac{\partial(u(x,t))}{\partial t}=\sum_{n=1}^{\infty}c_n\sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}\left(\frac{n\pi a}{l}\right)$$

$$\frac{\partial}{\partial t}l(x,0)\sum_{n=1}^{\infty}C_n\frac{n\pi a}{l}\sin\frac{n\pi x}{l}$$

From (5) & above result

$$\sin^3\frac{\pi x}{l}=\sum_{n=1}^{\infty}c_n\frac{n\pi a}{l}\sin\frac{n\pi x}{l}$$

$$\frac{3}{4}\sin\frac{\pi x}{l}-\frac{1}{4}\sin\frac{3\pi x}{l}=\sum_{n=1}^{\infty}c_n\frac{n\pi a}{l}\sin\frac{n\pi x}{l}$$

$$=\left[c_1\frac{\pi a}{l}\sin\frac{\pi x}{l}+c_2\frac{2\pi a}{l}\sin\frac{2\pi x}{l}+-----\right]-\left[l/2\frac{l}{n\pi}\left(-\cos\frac{n\pi}{2}\right)-\frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$=\frac{4k}{l^2}2\frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$=\frac{8k}{n^2\pi^2}\sin\frac{n\pi}{2}$$

If $n=2m$ (an even number) $c_{2m}=0$

If $n=2m+1(an\ odd\ number), c_{2m+1}=\frac{8k}{(2m+1)^2\pi^2}(-1)^m$

Thus all c_n's are determined

Using

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)\ for\ 0 \le x \le l$$

$$D_n = \frac{2}{n\pi c}\int_0^l g(x)\sin\frac{n\pi x}{l}\,dx$$

$$= 0 \qquad\qquad \text{Since g (x) = 0}$$

Hence, $u(x,t) = \dfrac{8k}{\pi^2}\sum_{m=0}^{\infty}\dfrac{(-1)^m}{(2m+1)^2}\sin\dfrac{(m+1)\pi ct}{l}\sin\dfrac{(2m+1)\pi x}{l}$

1. Solve the boundary value problem

$u_{tt} = a^2 u_{xx}; 0 < x < l; t > 0\ with\ u(0,t) = 0, u(l,t) = 0\ \&\ u(x,0) = 0, u_t(x,0) = \sin^3\left(\dfrac{\pi x}{l}\right)$

Ans. $u(x,t)$ is the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} \rightarrow (1)$$

Given conditions are

$$u(0,t) = 0\forall t \rightarrow (2)\ and\ \mu_t(x,0) = \sin^3\frac{\pi x}{l}\forall x \in [0,l] \rightarrow (5)$$

$$u(l,t) = 0\forall t \rightarrow (3)$$

$$u(x,0)\forall 0 \le x \le l \rightarrow (4)$$

Comparing the coefficients of like terms,

$$c_1\frac{\pi a}{l} = \frac{3}{4}, c_2 = 0, c_3\left(\frac{3\pi a}{l}\right) = \frac{-1}{4}, c_4, c_5 --- c_n = 0$$

$$\Rightarrow c_1 = \frac{3l}{4\pi a}, c_2 = 0, c_3 = \frac{-1}{1/2\pi a}, c_4 = 0$$

Hence, satisfying the values in (9)

$$u(x,t) = \frac{3l}{4\pi a}\sin\frac{\pi x}{l}\sin\frac{\pi at}{l} - \frac{1}{12\pi a}\sin\frac{3\pi x}{l}\sin\frac{3\pi at}{l}$$

2. If a string of length l is initially at rest in equillibrium position and each of its points is given the velocity $V_o\sin^3\dfrac{\pi x}{l}$, find the displacement $y(x,t)$

Ans. with the explained notation, the displacement $y(x,t)$ is given by

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} \to (1)$$
$$y(0,t) = 0 \forall t \to (2)$$
$$y(l,t) = 0 \forall t \to (3)$$
$$y(x,0) = 0 \le x \le l \to (4)$$
$$\left.\frac{\partial y}{\partial t}\right|_{at\, t=0} = V_0 \sin^3 \frac{\pi x}{l} \to (5)$$

The most general solution of (1) satisfying (2) & (3) is

$$y(x,t) = \sum_{n=1}^{\infty}\left(C_n \cos\frac{n\pi ct}{l} + D_n \sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi x}{l} \to (6)$$

Using (4) we get $\sum C_n \sin\frac{n\pi x}{l} = 0 \forall x \in [0,l]$ which implies $C_n = 0$ for all n

Now, using (5), we get

$$\sum D_n \frac{n\pi c}{l}\sin\frac{n\pi x}{l} = V_0 \sin^3\frac{\pi x}{l}$$
$$= V_0\left[\frac{3}{4}\sin\frac{\pi x}{l} - \frac{1}{4}\sin\frac{3\pi x}{l}\right]$$

Hence $D_1 = \frac{-3l}{4\pi c V_o}, D_3 = \frac{-lV_0}{12\pi c}$

Hence $y(x,t) = \frac{-3lV_o}{4\pi c}\sin\frac{\pi ct}{L}\sin\frac{\pi x}{L} - \frac{lV_0}{12\pi c}\sin\frac{3\pi ct}{l}\sin\frac{\pi x}{l}$

Fill in the blanks:

1. If the number of arbitary constants to be eliminated is equal to the number of independent variables then we get a partial differential equation of ____________ order
2. If the number of arbitary constants to be eliminated is greater than the number of independent variables then we get a partial differential equation of ____________ order
3. The partial differential equation by eliminating the arbitary constants from $z = ax + by$ is __________
4. The partial differential equation by eliminating the arbitary constants from $z = ax^2 + by^2$ is

5. The partial differential equation by eliminating the arbitary constant from $z=(x+a)(y+b)$ is_____________

6. The partial differential equation by eliminating the arbitary constants from $z=(x^2+a)(y^2+b)$ is___________

7. The partial differential equation by eliminating the arbitary constants from $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ is__________

8. The partial differential equation of all spheres whose centers lie on the z-axis is ________
9. The partial differential equation by eliminating arbitary function from $z=f(x^2+y^2)$ is___________
10. The partial differential equation by eliminating arbitary function from $z=f(x^2-y^2)$ is__________
11. The partial differential equation by eliminating arbitary function from $z=x^n f(y/x)$ is__________
12. The partial differential equation by eliminating arbitary function from $z=y f(y/x)$ is__________
13. The partial differential equation by eliminating the arbitary function from the relation $z=f(\sin x+\cos y)$ is __________
14. The partial differential equation by eliminating the arbitary function from the relation $z=y^2+2f\left(\frac{1}{x}+\log y\right)$ is __________
15. The general solution of $2p+3q=1$ is _________
16. The general solution of $xp+yq=3z$ is ________
17. The general solution of $ptanx+q\tan y=\tan z$ is ________
18. The general solution of $yzp-xzq=xy$ is _________
19. The general solution of $\sqrt{p}+\sqrt{q}=1$ is _________
20. The general solution of $p^3-q^3=0$ is __________
21. The general solution of $pq+p+q=0$ is __________
22. The general solution of $p^2+q^2=m^2$ is __________
23. The general solution of $p^2-q^2=4$ is ___________
24. General form of Clairauti equation is__________
25. The general solution of $z=px+qy+f(p,q)$ is ___________
26. The general solution of $z=px+qy+\log pq$ is ____________
27. The general solution of $z=px+qy+pq$ is _____________
28. The general solution of $z=px+qy+p^2q^2$ is ______________
29. The general solution of $z=px+qy+\sqrt{1+p^2+q^2}$ is __________

30. The general solution of $(p-q)(z-px-qy)=1$ is __________
31. The general solution of $z=px+qy-2p-3q$ is ____________
32. The general solution of $z=px+qy-2pq$ is __________
33. The general solution of $z=px+qy+p/q$ is __________
34. The general solution of $z=px+qy+3\sqrt{pq}$ is ___________
35. By eliminating a & b from $z=a(x+y)+b$, the partial differential equation is __________
36. By eliminating a & b from $z=ax+by+a^2+b^2$, the partial differential equation formed is _________
37. By eliminating a & b from $z=ax+by+a/b$, the partial differential equation formed is ___________
38. By eliminating a & b from $z=(x-a)^2+(y-b)^2+1$, the partial differential equation formed is ___________
39. By eliminating a & b from $z=ax^3+by^3$, the partial differential equation formed is ____________
40. The general solution of $p=q^2$ is __________
41. The general solution of $pq=4$ is __________
42. The general solution of $p^2+q^2=4$ is _______
43. The general solution of $px=qy$ is __________
44. The general solution of $pe^y=qe^x$ is _________
45. The general solution of $p^2+q^2=x+y$ is ______
46. The general solution of $z=pq$ is _____________
47. The general solution of $dx=dy=dz$ is _________
48. The general solution of $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ is _________