**R18**

Code No:151AA

# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

## B .Tech I Year I Semester Examinations, December - 2018

## MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, PTM)

**Time: 3 hours** **Max. Marks: 75**

**Note:** This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

## PART - A

**(25 Marks)**

1.a) If A is Hermitian matrix and B is a Skew- Hermitian matrix , prove that (B+iA) is Skew -Hermitian matrix. [2]

b) Let A be a square matix of order 3 with Eigenvalues 2, 2 and 3 and A is diagonalizable then find rank of (A-2I). [2]

c) State Cauchy's root test. [2]

d) Find the value of $\Gamma\left(-\frac{1}{2}\right)$ [2]

e) Verify Euler's theorem for the function $xy+yz+zx$ . [2]

f) Prove that the transpose of a unitary matrix is unitary. [3]

g) Find the Eigen values of the matrix $A=\begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$ [3]

h) Test for convergence $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$ . [3]

i) Discuss the applicability of Rolle's Theorem to the function $f(x)=2+(x-1)^{\frac{2}{3}}$ in the interval [0, 2] . [3]

j) If $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\frac{y}{x}$ show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$. [3]

## PART - B

**(50 Marks)**

2.a) Reduce the given matrix into normal form and hence find the rank

$$\begin{pmatrix} 2 & 3 & -2 & 5 & 1 \\ 3 & -1 & 2 & 0 & 4 \\ 4 & -5 & 6 & -5 & 7 \end{pmatrix}$$

b) Solve the equations $x+y+z=6;\ 3x+3y+4z=20;\ 2x+y+3z=13$ using Gauss elimination method. [5+5]

**OR**

3.a) Find the rank of the matrix $\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{pmatrix}$ by reducing it to Normal form.

b) Solve the system of equations by Gauss-Seidel method $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. [5+5]

4.a) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, and $A^{-1} = \alpha A^2 + \beta A + \gamma I$, $\alpha, \beta, \gamma \in R$, then find $\alpha + \beta + \gamma$.

b) Find the nature of the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$. [5+5]

**OR**

5.a) Let A be a 3×3 matric over R such that det (A) =6 and tr (A)=0. If det(A+I)=0, where I is the identity matrix of order 3, then find the Eigen values of A .

b) Reduce the quadratic form $5x^2 + 26y^2 + 10z^2 + 4yz + 14zx + 6xy$ to canonical form. [5+5]

6.a) Test the convergence of the series $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \ldots$.

b) Examine the following series for convergence $\sum \frac{(-1)^{n-1} \sin nx}{n^3}$. [5+5]

**OR**

7.a) Test for convergence of the series $\sum \frac{x^n}{(2n)!}$.

b) Examine for absolute convergence the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$. [5+5]

8.a) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$, $o < x < \frac{\pi}{2}$.

b) Find the surface area of the solid generated by revolving the loop of the curve $9y^2 = x(x-3)^2$. [5+5]

**OR**

9.a) Show that $|\cos b - \cos a| \le |b - a|$.

b) Show that $\int_0^{\infty} x^{m-1}(a-x)^{n-1}\,dx = a^{m+n-1}\beta(m,n)$. [5+5]

10.a) If $u = f(y-z, z-x, x-y)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

b) If $x = uv, y = \frac{u+v}{u-v}$ determine $\frac{\partial(u,v)}{\partial(x,y)}$. [5+5]

**OR**

11.a) If $U = x + y - z, V = x - y + z, W = x^2 + y^2 + z^2 - 2yz$, show that the functions are functionally dependent.

b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. [5+5]

---ooOoo---