

Time: 3 Hours

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART - A

(25 Marks)

- 1.a) Find the negation of $p \rightarrow q$. [2]
- b) Test the validity of the following argument
 $p \wedge r \rightarrow \neg q, \neg q \rightarrow r \therefore p \wedge r \rightarrow r$ [3]
- c) If $f(x) = x^2 - 6 = y$, then find $f^{-1}(y)$. [2]
- d) If $f: G_1 \rightarrow G_2$ is a homomorphism and $a \in G$ then prove that $[f(a)]^{-1} = f(a^{-1})$. [3]
- e) How many 5 digit numbers are possible, which are greater than 40000 with the digits 1, 2, 3, 4, 5. [2]
- f) Find the number of positive integer solutions of $x + y + z = 12$. [3]
- g) Solve the recurrence relation $u_{n+2} - u_{n+1} - 6u_n = 0$. [2]
- h) Find the generating function of the sequence 1, 3, 3², 3³, [3]
- i) If the adjacency matrix of the Graph is $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$, then draw the graph. [2]
- j) If G is a k regular graph with 18 edges and the order of the graph is 9. Find the value of k. [3]

PART - B

(50 Marks)

- 2.a) Test the validity of the following argument.
If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
Therefore I must watch TV in the evenings.
Prove that the following argument is valid. [5+5]
- b) $\neg \exists x(p(x) \wedge q(x))$
 $p(a)$
 $\therefore \neg q(a)$

OR

3.a) Prove that $(p \uparrow q) \rightarrow r$ and $(p \wedge q) \vee r$ are logically equivalent.

b) Prove that the following argument is valid.

$$\forall x p(x) \rightarrow \neg q(x)$$

$$\neg \exists x ((r(x) \vee s(x)) \wedge \neg q(x))$$

$r(a)$

$$\therefore \neg p(a)$$

[5+5]

4.a) Let $X = \{1, 2, 3\}$ and f, g, h and s be functions from X to X given by $f = \{(1, 2), (2, 3), (3, 1)\}$, $g = \{(1, 2), (2, 1), (3, 3)\}$ $h = \{(1, 1), (2, 2), (3, 1)\}$ Find $f \circ g$, $f \circ h \circ g$.

b) If $f : G_1 \rightarrow G_2$ is an isomorphism, then prove that $f^{-1} : G_2 \rightarrow G_1$ is also an isomorphism.

[5+5]

OR

5.a) Prove that the relation 'a congruent to b mod H' is an equivalence relation.

b) Prove that the set of even integers forms a group under addition.

[5+5]

6.a) Find the number of solutions of $x_1 + x_2 + x_3 = 19$ with the condition $x_1 > 1, x_2 > 2, x_3 > 1$.

b) Prove that if 11 integers are selected from among $\{1, 2, \dots, 20\}$, then the selection includes integer a and b such that $a - b = 2$.

[5+5]

OR

7.a) Find the number of integers < 250 and divisible by 3 or 5 or 11.

b) Suppose 14 students in a class appear at a university examination. Prove that there exists at least two among them whose seat number differ by a multiple of 13.

[5+5]

8. Solve the recurrence relation. $u_n - 2u_{n-1} - 3u_{n-2} = 5^n, n \geq 2, u_0 = 1, u_1 = 1$

[10]

OR

9. Solve the recurrence relation using generating function. $u_{n+2} - 2u_{n+1} + u_n = 2^n$

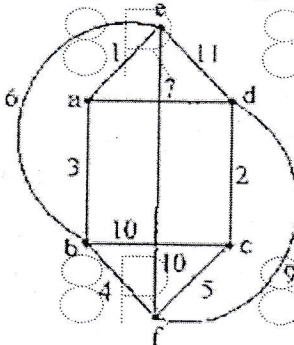
$$u_0 = 2, u_1 = 1$$

[10]

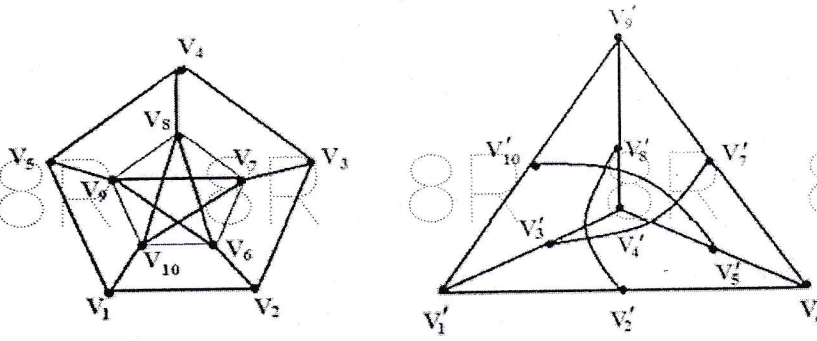
10.a) Suppose that G is a non directed graph with 12 edges. Suppose that G has 6 vertices of degree 3 and the rest have degree less than 3. Determine the minimum number of vertices G can have.

b) Find the minimal spanning tree using Krushal's algorithm.

[5+5]



11.a) Show that the following graphs are isomorphic. **OR**



b) Prove that A graph G with at least one edge is 2-chromatic if and only if G has no cycle of odd length. [5+5]

---ooOoo---