

Code No: 133BD

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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2017

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(25 Marks)

- 1.a) Show that $f(z) = z + \bar{z}$ is not analytic any where in the complex plane. [2]
- b) Write Cauchy-Riemann equations in Polar form. [3]
- c) Find the residues at the poles of the function $f(z) = \frac{1}{(z+1)(z+2)}$. [2]
- d) Expand $f(z) = \tan z$ in Taylor's series about the point $z=0$. [3]
- e) Define Bilinear transformation. [2]
- f) Define for a complex function: i) Isolated Singularity ii) Removable Singularity. [3]
- g) If $f(x) = x^2$ in $[-\pi, \pi]$, find a_0 in Fourier series. [2]
- h) State Fourier integral theorem. [3]
- i) Write the one dimensional Heat equation in steady state. [2]
- j) Classify partial differential equation $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$. [3]

PART-B

(50Marks)

- 2.a) Determine p so that the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + \tan^{-1}(\frac{px}{y})$ is analytic.
 - b) Find the analytic function $f(z) = u + iv$ if $u - v = e^x [\cos y - \sin y]$. [5+5]
- OR
- 3.a) Determine the analytical function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 = 1$. Also find the harmonic conjugate of this real part.
 - b) Prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |f(z)|^2 = 4 |f'(z)|^2$. [5+5]
- 4.a) Using Cauchy integral formula, find $\int_C \frac{e^{2z}}{(z+1)^3} dz$, where C is the curve $|z| = 2$.
 - b) Evaluate $\int (x^2 - iy^2) dz$ along a straight line from $(0,0)$ to $(0,1)$ and then from $(0,1)$ to $(2,1)$. [5+5]

OR

5. Find Laurent's series of $\frac{z}{(z-1)(z-2)}$ about:

- a) $|z| < 1$
- b) $|z| > 1$
- c) $1 < |z| < 2$

[10]

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6. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$. [10]

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7. Find Mobius transformation that maps:
a) $0, 1, \infty$ into $-5, -1, 3$, Find fixed points.
b) $\infty, i, 0$ to $0, -i, \infty$. [5+5]

8. Find the Fourier series of $f(x) = e^{-x}$ in the interval $(0, 2\pi)$. Hence, deduce that
 $\frac{\pi}{2} \frac{1}{\sinh \pi} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+1}$ [10]

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9.a) Find the Fourier series for $f(x) = x + x^2$ in $-\pi < x < \pi$.
b) Find the Fourier cosine transform of e^{-x^2} [5+5]

10. Solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, $u(0, y) = 8e^{-3y}$ by the method of separation of variables. [10]

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11. The ends of a uniform string of length $2l$ are fixed. The initial displacement is $y(x, 0) = 3x(2l - x)$, $0 < x < 2l$, while the initial velocity is zero, Find the displacement at any distance x from the end $x=0$ at any time 't'. [10]

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