R13

Code No: 111AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, October/November - 2016

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A (25 Marks)

1.a) Define Eigen vector of a matrix. [2]

b) Write the working procedure to solve the system of non-homogenous equations. [3]

c) Verify for $x=u,\ y=u\tan v,\ z=w,\ J\left(\frac{x,y,z}{u,v,w}\right)\times J'\left(\frac{u,v,w}{x,y,z}\right)=1.$ [2]

d) Give an example of a function that is continuous on [-1, 1] and for which mean value theorem does not hold, explain. [3]

e) Show that $\beta(p,q)=\beta(p+1,q)+\beta(p,q+1).$ [2]

f) Evaluate $\int_0^1\int_1^{2-x} xy\,dx\,dy.$ [3]

g) Explain the method of solving Bernoulli equation. [2]

h) Solve $(D^4+2D^2n^2+n^4)y=0.$ [3]

i) State and prove change of scale property of Laplace transforms. [2]

j) Prove that $L^{(-1)}\{F(s)\}=f(t)$ and $f(0)=0$ then $L^{(-1)}\{sF(s)\}=\frac{df}{dt}.$ [3]

PART-B (50 Marks)

2. Determine a non-singular matrix P such that P^TAP is a diagonal matrix, where

$$A=\begin{bmatrix}0&1&2\\1&0&3\\2&3&0\end{bmatrix}$$ [10]

OR

3.a) Show that the two matrices A, $C^{-1}AC$ have the same latent roots.

b) For a matrix $A=\begin{bmatrix}1&2&-3\\0&3&2\\0&0&-2\end{bmatrix}$ find the Eigen values of $3A^3+5A^2-6A+2I.$ [5+5]

4.a) Find the minimum and maximum values of $\sin x+\sin y+\sin(x+y).$

b) If $u=\frac{1}{\sqrt{x^2+y^2+z^2}},\ x^2+y^2+z^2\neq 0$ then evaluate $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}.$ [5+5]

OR

5.a) Prove that $\frac{\pi}{6}+\frac{1}{5\sqrt{3}}<\sin^{-1}\left(\frac{3}{5}\right)<\frac{\pi}{6}+\frac{1}{8}$.

b) Verify Lagrange's mean value theorem for $f(x)=\begin{cases} x\sin\frac{1}{x} & (x\neq 0) \\ 0 & (x=0) \end{cases}$ in [-1. 1]. [5+5]

6.a) Evaluate $\iiint_V x^{l-1}y^{m-1}z^{n-1}\,dxdydz$ where V is the region $x\geq 0,\ y\geq 0,\ z\geq 0$ and the plane $x+y+z<1$.

b) Express the integral $\int_0^\infty \frac{x^c}{c^x}dx\,(c>1)$ in terms of Gamma function. [5+5]

OR

7.a) By changing the order of integration and evaluate $\int_0^b\int_0^{\frac{a\sqrt{b^2-y^2}}{b}} xydydx$.

b) Find the area enclosed by the parabolas $x^2=y$ and $y^2=x$. [5+5]

8.a) The number N of bacteria in a culture grows at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hour?

b) Solve $(x-y)dx-dy=0,\ \ y(0)=2$. [5+5]

OR

9. Solve $(D^2-4D+4)y=x^2\sin x+e^{2x}+3$. [10]

10.a) State and prove convolution theorem for Laplace transforms.

b) Find the Laplace transform of $f(t)=|t-1|+|t+1|,\ t\geq 0$. [5+5]

OR

11.a) Solve the differential equation using Laplace transforms

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=e^{-t};\ x(0)=0,\ x'(0)=1.$$

b) Evaluate $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$. [5+5]

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