Code No: 113AH **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, December-2014

MATHEMATICS-III

(Common to EEE, ECE, EIE, AGE)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

Part- A **(25 Marks)**

1.a) Find the complementary function of $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=x^3\log x.$ [2M]

b) Find the singular points of the differential equation:
$x^3(x-1)\frac{d^2y}{dx^2}+2(x-1)\frac{dy}{dx}+y=0.$ [3M]

c) Write the value of $J_{-1/2}(x)$. [2M]

d) Obtain the value of $P_3(x)$. [3M]

e) Determine the region in the z-plane represented by $\frac{\pi}{3}<\text{amp}(z)<\frac{\pi}{2}$. [2M]

f) State Cauchy's integral theorem. [3M]

g) Define an essential singularity [2M]

h) Expand **cosz** in Taylor's series about the point $z=\pi/2$. [3M]

i) Define conformal transformation. [2M]

j) Find the invariant points of the transformation $w=(z-1)/(z+1)$. [3M]

Part-B **(50 Marks)**

2.a) Solve $(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x.$

b) Solve the equation $y''+x^2y=0$ in series.

OR

3.a) Solve $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+\frac{y}{x^2}=\frac{\log x}{x^2}.$

b) Solve the equation $3x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}-y=0$ in power series.

4.a) Express $f(x)=x^4+3x^3-x^2+5x-2$ in terms of Legendre polynomials.

b) Prove that $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x).$

c) Prove that $\frac{d}{dx}J_0(x)=-J_1(x).$

OR

5.a) State and prove the generating function for $P_n(x)$.

b) Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.

c) If $J_{n+1}(x) = \frac{2}{x} J_n(x) - J_0(x)$, then find the value of n.

6.a) Show that the real and imaginary parts of an analytic function are harmonic.

b) Evaluate $\int_C |z| dz$, where C is the contour consisting of the straight line from $z = -i$ to $z = i$.

c) Evaluate $\oint_C \frac{e^z}{(z+1)^2} dz$, where C is $|z-1| = 3$.

OR

7.a) Show that the function $f(z) = \bar{z}$ is not an analytic function at any point.

b) If the potential function is $\log(x^2 + y^2)$, find the flux function and the complex potential function.

c) Evaluate $\int_C \frac{z^2 - z + 1}{z-1} dz$, where C is the circle $|z| = 1/2$.

8.a) Find the Laurent's series expansion of $f(z) = \frac{7z^2 - 9z - 18}{z^3 - 9z}$ in the regions $|z| > 3$ and $0 < |z-3| < 3$.

b) Apply the calculus of residues to prove that $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1} = \frac{\pi}{\sqrt{2}}$.

OR

9.a) State and prove the Residue theorem.

b) Evaluate $\int_0^{\infty} \frac{\cos ax}{x^2 + 1} dx$.

10.a) Find the bilinear transformation which maps **1, i, -1 to 2, i, -2** respectively. Find the fixed and critical points of the transformation.

b) Show that under the transformation $w = 1/z$, a circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in the w-plane.

OR

11.a) Show that the condition for transformation $w = (az+b)/(cz+d)$ to make the circle $|w| = 1$ correspond to a straight line in the z-plane is $|a| = |c|$.

b) Discuss the transformation $w = z^2$.

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