Code No: 111AB **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year Examinations, December-2014/January-2015

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

Part- A (25 Marks)

1.a) Define Hermitian matrix and give a suitable example. [2m]

b) Write any three properties of Eigen values. [3m]

c) State Cauchy's mean value theorem. [2m]

d) Find the points on the surface $z^2 = xy+1$ nearest to the origin. [3m]

e) Sketch (roughly) the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. [2m]

f) Express $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$ in terms of gamma functions. [3m]

g) Formulate the differential equation by eliminating the constants from the equation: $xy = Ae^x + Be^{-x}$. [2m]

h) Solve $(D^2+5D+6)y = e^x$. [3m]

i) Find $L^{-1}\left(\dfrac{1}{s(s+2)^3}\right)$. [2m]

j) Find $L\left(te^{-t}\cosh t\right)$. [3m]

Part-B (50 Marks)

2.a) Show that $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$ is unitary and find A^{-1}.

b) Find the values of a and b for which the equations:
$x + ay + z = 3$; $x + 2y + 2z = b$; $x + 5y + 3z = 9$ are consistent. When will these equations have a unique solution?

OR

3.a) If $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$, then show that $(I-A)(I+A)^{-1}$ is a unitary matrix.

b) Reduce the quadratic form $8x^2+7y^2+3z^2-8yz+4zx-12xy$ to the canonical form and specify the matrix of transformation.

4.a) Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, if $0 < a < b < 1$. Hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

OR

5.a) If $f(x) = \sin^1 x$, $0 < a < b < 1$, use Mean value theorem to prove that $\frac{b-a}{\sqrt{(1-a^2)}} < \sin^{-1} b - sin^{-1} a < \frac{b-a}{\sqrt{(1-b^2)}}$.

b) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Do u and v functionally related? If so, find the relationship between u and v.

6.a) Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$, where n is a positive integer and m>-1. Hence evaluate $\int_0^1 x(\log x)^3 dx$.

b) Evaluate $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \frac{x\,dydx}{\sqrt{(x^2+y^2)}}$ by changing its order of integration.

OR

7.a) Evaluate $\int_0^2 \int_0^{\sqrt{(2x-x^2)}} \frac{xdydx}{(x^2+y^2)}$ by changing to polar coordinates.

b) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

8.a) Apply the method of variation of parameter to solve $\frac{d^2y}{dx^2} + 9y = \tan 3x$.

b) Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$.

OR

9.a) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$.

b) A body is originally at 80^0C and cools down to 60^0C in 20 minutes. If the temperature of the air is 40^0C, find the temperature of the body after 40 minutes.

10.a) Apply Convolution theorem to evaluate $L^{-1}\left(\frac{s}{(s^2+1)(s^2+4)}\right)$.

b) Solve $y'' + 2y' + 5y = e^t \sin t$, $y(0) = 0$, $y'(0) = 1$ by transform method.

OR

11.a) Find (i) $L(\sinh 3t \cos^2 t)$ and (ii) $\sin 2t\, \delta(t-3)$.

b) Solve $ty'' + 2y' + ty = \sin t$, $y(0) = 1$, by applying Laplace transform method.

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