

B.Tech II Year - II Semester Examinations, April/May-2012

MATHEMATICS-III

(COMMON TO AE, EEE, ECE, EIE, MMT)

Time: 3 hours

Max. Marks: 80

Answer any five questions
All questions carry equal marks

- - -

- 1.a) Evaluate $\int_0^1 \frac{x^4 dx}{\sqrt{1-x^2}}$ using Beta and Gamma functions.
- b) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [16]
- 2.a) Express $x^3 - 2x^2 + 3x + 2$ in terms of Legendre polynomial.
- b) Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$. [16]
- 3.a) Find an analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$.
- b) Find all solutions of $\cos z = \frac{1}{2}$. [16]
- 4.a) Evaluate $\int_{(0,0)}^{(1,1)} [(x^2 + 2xy) + i(y^2 - x)]dz$ along $y = x^2$.
- b) Evaluate $\int_c \frac{dz}{e^z(z-1)^3}$ where $|z| = 2$. [16]
5. State and prove Laurent's theorem. [16]
- 6.a) Find the poles and the corresponding residues of $\frac{\sin^2 z}{(z - \frac{\pi}{6})^2}$.
- b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{(5 + 4\cos\theta)}$. [16]
- 7.a) Evaluate $\int_0^\infty \frac{dx}{(1+x^4)}$.
- b) Use Rouché's theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disc $|z| < 3/2$ and four roots in $3/2 < |z| < 2$. [16]
- 8.a) Find the image of the rectangular region $-1 \leq x \leq 3, -\pi \leq y \leq \pi$ under the transformation $w = e^z$.
- b) Find the bilinear transformation which maps the points $\infty, i, 0$ of z -plane to $0, i, \infty$ in w plane. [16]

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