09 - 1st year - All subjects.

Code No: 51002 **R09**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, May - 2016

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, CHEM, EIE, BME, IT, MCT, ETM, MMT, AE, BT, AME, MIE, PTE, MSNT, AGE)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

1.a) i) Show that the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for p > 1 and diverges for $p \le 1$.

ii) Discuss the convergence of $\sum \frac{(n!)^2}{(2n)!}x^{2n}$.

b) Define absolute and conditional convergence. Test whether the series $1+\frac{1}{2^2}-\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}-\frac{1}{7^2}-\frac{1}{8^2}+\ldots$ is convergent or not. [7+8]

2.a) Verify the results of Cauchy's mean value theorem for the function $\sin x$ and $\cos x$ in the interval $[a,b]$.

b) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm. [7+8]

3.a) Show that the evolute of the cycloid $x = a(\theta - \sin\theta);\ y = a(1-\cos\theta)$ is another equal cycloid.

b) Trace the curve $y^2(a-x) = x^2(a+x)$. [7+8]

4.a) Find the area of the segment cut off from the parabola $x^2 = 8y$ by the line $x - 2y + 8 = 0$.

b) Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate it. [7+8]

5.a) Solve $\frac{dy}{dx} + y\tan x = y^3\sec x$.

b) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, where λ is the parameter. [7+8]

6.a) Solve, by the method of variation of parameters, $y'' - 2y' + y = e^x \log x$.

b) The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$. Find the current i as a function of t given that I is the maximum current, and $i = 0$ when $t = 0$. [7+8]

7.a) Find the Laplace transform of i) $\frac{1-e^t}{t}$ ii) $\int_0^\infty te^{-2t}\sin t\,dt$.

b) Using Laplace transform, solve $y'' + y = t,\ y(0) = 1,\ y'(0) = -2$. [7+8]

8.a) If $\overline{F} = (y+z)\overline{i} + (z+x)\overline{j} + (x+y)\overline{k}$ be a vector function, show that $\overline{F}$ is irrotational and find the potential function.

b) Using Gauss divergence theorem evaluate $\int_C \overline{F}\cdot\overline{n}\,ds$ where $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$ taken over the rectangular parallelepiped bounded by the coordinate planes and $x = a,\ y = b,\ z = c$. [7+8]

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