Code No.: MA101BS/151AA | R20 | H.T.No. | | | 8 | R | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Supply) -January- 2025
LINEAR ALGEBRA AND CALCULUS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 70]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A (20 Marks)

1. a) Define symmetric matrix and give an example. [2M]
   b) State the conditions when the system of homogeneous equations $AX = 0$ will have (i) Trivial solution (ii) non trivial solutions. [2M]
   c) If the Eigen values of A are 1,2,3 then find (i) det A (ii) Trace of A. [2M]
   d) State Cayley- Hamilton theorem. [2M]
   e) State Raabe's test. [2M]
   f) Test for convergence $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$. [2M]
   g) Verify whether Rolle's Theorem can be applied to $f(x) = \tan x$ in $[0,\pi]$. [2M]
   h) Evaluate $\int_0^1 x^5(1-x)^3 dx$. [2M]
   i) Verify if $\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$ if $f(x,y) = x^3 + y^3 - 3axy$. [2M]
   j) Define saddle point. [2M]

PART-B (50 Marks)

2.a) Find the Rank of the matrix by reducing into Echelon Form. [5M]

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

b) For what values of λ and μ the system of equations [5M]
$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have
i) no solution ii) a unique solution iii) an infinite number of solution

OR

3. Solve the following system of equations using Gauss-Seidel iteration method. $10x + y + z = 12, 2x + 10y + z = 13, 2x + 2y + 10z = 14$. [10M]

4. Find the Eigen values and the corresponding Eigen vectors of the matrix. [10M]

$$A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

OR

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$ to canonical form by Orthogonal transformation and hence discuss the nature, rank, index and signature of the quadratic form. [10M]

6. Examine the convergence of the series [10M]

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\ldots\ldots:x>0$$

OR

7.a) Find the nature of the series $\sum\frac{(n!)^2}{(2n)!}.x^{2n}$ (x>0) [5M]

b) Test for convergence $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n(n+1)(n+2)}}$ [5M]

8. If a<b, Prove that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$ using Lagrange's mean value theorem and Hence deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$ [10M]

OR

9. Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where m > 0, n > 0. [10M]

10. Show that functions u =xy + yz + zx, v= $x^2+y^2+z^2$ and w=x + y + z are functionally related. Find the relation between them. [10M]

OR

11.a) A rectangular box open at the top is to have volume of 32cubic ft. Find the dimensions of the box requiring least material for its construction. [7M]

b) Find three positive numbers whose sum is 100 and whose product is maximum. [3M]

****