28/01/25

Code No.: MA201BS | R20 | H.T.No. 8 R

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–II–Semester End Examinations (Supply) –January - 2025
DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS
(Common for all)

[Time: 3 Hours] [Max. Marks: 70]

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A (20 Marks)

1\. a) Define Exact differential equation. [2M]

b) Find the integrating factor of $\frac{dy}{dx} + 2xy = e^{-x^2}$. [2M]

c) Solve $(D^3 - 5D^2 + 8D - 4)y = 0$. [2M]

d) Find particular integral of $(D^2 + 5D + 6)y = e^x$. [2M]

e) Evaluate $\int_0^3 \int_0^2 dx\, dy$. [2M]

f) Evaluate $\int_{\theta=0}^{\pi} \int_{r=0}^{a\cos\theta} dr d\theta$. [2M]

g) Evaluate $\nabla(xyz)$. [2M]

h) Define Solenoidal vector and Irrotational vectors . [2M]

i) State Green's theorem. [2M]

j) Write the formula for Surface integral in YZ plane. [2M]

PART-B (50 Marks)

2\. A body kept in air with temperature 25^0C cools from 140^0C to 80^0C in 20 minutes. Find when the body cools down to 35^0C. [10M]

OR

3\.a) Solve $\frac{dy}{dx} + y\, tanx = y^2\, secx$. [5M]

b) Solve $\left(1 + e^{x/y}\right)dx + \left(1 - \frac{x}{y}\right)e^{x/y}dy = 0$. [5M]

4\. Solve $(D^2 + 1)y = Cosecx$ by the method of variation of parameters. [10M]

OR

5\. Solve the differential equation: [10M]

$$(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right).$$

6\. Evaluate $\iiint xy^2z\, dxdydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [10M]

OR

7\. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy$ dx dy and hence evaluate the same. [10M]

8.a) Find curl $\bar{f}$ at the point (1,2,3),given that [5M]

$\bar{f}$ = grad($x^3\,y + y^3\,z + z^3\,x - x^2\,y^2\,z^2$).

b) Show that div$(r^n\,\bar{r}\,) = (n+3)\,r^n$,where $r^2 = x^2 + y^2 + z^2$. [5M]

OR

9.a) Find the directional derivative of $2xy + z^2$ at (1,-1,3) in the direction of $\bar{\imath} + 2\,\bar{\jmath} + 3\,\bar{k}$. [5M]

b) Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)\,k$ is Irrotational and find its scalar potential. [5M]

10. Verify Stokes theorem for $\bar{F} = (2x-y)i - yz^2 j - y^2 z\,k$, over the upper half surface of the sphere $x^2+y^2+z^2=1$ bounded by the projection of the xy-plane. [10M]

OR

11. Verify Gauss's divergence theorem for $\bar{F} = 2x^2 y\bar{\imath} - y^2\bar{\jmath} + 4xz^2\bar{k}$ taken over the region of the first octant of the cylinder $y^2 + z^2 = 9$ and $x = 2$. [10M]

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