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CMR ENGINEERING COLLEGE: : HYDERABAD

UGC AUTONOMOUS

I-B.TECH-I-Semester End Examinations (Regular) - February- 2024

MATRIX ALGEBRA AND DIFFERENTIAL EQUATIONS

(Common for all)

[Time: 3 Hours]

[Max. Marks: 60]

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 10 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(10 Marks)

1. a) Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 2 & 2 \end{pmatrix}$. [1M]
- b) Find the value of 'k' such that the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{pmatrix}$ is 2. [1M]
- c) Find the product of the Eigen values of $A = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. [1M]
- d) Find the symmetric matrix associated with the quadratic form $2xy + 2yz + 2zx$. [1M]
- e) State Cauchy's Mean Value theorem. [1M]
- f) If $u = x + y$ and $v = xy$ then find $\frac{\partial(u,v)}{\partial(x,y)}$. [1M]
- g) State the condition for exactness of the differential equation $M(x,y)dx + N(x,y)dy = 0$. [1M]
- h) Find the integrating factor of $\frac{dy}{dx} - \frac{2y}{x} = \frac{5x^2}{(2+x)(3-2x)}$. [1M]
- i) Find the complementary function of $(D^2 + 16)y = \cos 2x$. [1M]
- j) Find the Particular integral of $(D^2 - 3D + 2)y = e^{5x}$. [1M]

PART-B

(50 Marks)

2. a) Reduce the matrix $\begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{pmatrix}$ into normal form and hence find its rank. [5M]
- b) Solve the following system by Gauss elimination method [5M]
 $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$.

OR

3. Find an LU decomposition of the matrix A and solve the linear system [10M]
 $x + y + z = 1, 3x + y - 3z = 5, x - 2y - 5z = 10$.
4. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$, Hence find A^{-1} and A^4 . [10M]

OR

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2xz$ to canonical form and [10M]
hence state nature, rank, index, and signature of the quadratic form.

the temperature be $25^{\circ}C$.

10. Solve $(D^2 + 3D + 2)y = xe^x \sin x$. [10M]

OR

11. Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$. [10M]

6. Using Lagrange's mean value theorem ($0 < a < b < 1$) Prove that [10M]

$\frac{b-a}{(1+b^2)} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{(1+a^2)}$ and hence deduce the following.

(i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \left(\frac{4}{3} \right) < \frac{\pi}{4} + \frac{1}{6}$ (ii) $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$

OR

7. Find the point on a plane $x + 2y + 3z - 4 = 0$, which is nearest to the origin. [10M]

8. a) Solve the differential equation $(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$. [5M]

b) Solve the differential equation $(x + y + 1) \frac{dy}{dx} = 1$. [5M]

OR

9. The temperature of the body drops from $100^{\circ}C$ to $75^{\circ}C$ in 10 minutes when the surrounding air is at $20^{\circ}C$. What will be the temperature after half an hour. When will the temperature be $25^{\circ}C$. [10M]

10. Solve $(D^2 + 3D + 2)y = xe^x \sin x$. [10M]

OR

11. Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$. [10M]
