Code No.: MA101BS

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CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Supply) -February- 2024
LIN EAR ALGEBRA AND CALCULUS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 70]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(20 Marks)**

1. a) Define rank of a matrix. [2M]
b) Show that the vectors $X_1=(1,1,1)$ $X_2=(3,1,2)$ and $X_3=(2,1,4)$ are linearly independent. [2M]
c) Find the sum and product of the Eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ [2M]
d) Define Rank, Index and signature of Quadratic form. [2M]
e) State Cauchy's root test. [2M]
f) Examine the convergence of the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}$............ [2M]
g) Prove that B (m, n) = B (n, m). [2M]
h) State Cauchy's Mean value theorem [2M]
i) Write any two properties of Jacobian [2M]
j) If f(x, y) = xy+(x-y) then find the stationary points. [2M]

PART-B **(50 Marks)**

2. a) Find the Rank of the matrix by reducing $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into Normal form [7M]
b) State the conditions when the system of non-homogeneous equations $AX = B$ will have (i) Unique solution (ii) infinite no. of solutions (iii) no solution. [3M]

OR

3. Solve the following system of equations using Gauss-Seidel iteration method. [10M]

$$8x-3y+2z=20, 4x+11y-z=33, 6x+3y+12z=36.$$

4. Verify Cayley-Hamilton theorem and hence find $A^{-1} and A^4$ for A= $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ [10M]

OR

5. Reduce the quadratic form $2x^2+2y^2+2z^2-2xy-2xz-2yz$ to canonical form by Orthogonal transformation and hence discuss the nature, rank, index and signature of the quadratic form. [10M]

6. a) Test whether the series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$ is convergent. [5M]

b) Apply integral test to test the convergence of the series $\sum_{2}^{\infty}\frac{1}{n^2}\sin\left(\frac{\pi}{n}\right)$ [5M]

OR

7. Examine for absolute convergence the series. [10M]

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\dots\dots$

8. Verify Rolle's Theorem for the function $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in [a, b]. [10M]

OR

9. a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. [3M]

b) Find the volume of solid generated by the revolution of the area bounded by [7M]

$x^2 + y^2 = 2ax$ and $z^2 = 2ax$

10. a) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ [5M]

b) If $u = f(y - z, z - x, x - y)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ [5M]

OR

11. Examine for minimum and maximum values of $\sin x + \sin y + \sin(x+y)$. [10M]

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