Code No.: MA101BS | R22 | H.T.No. | | | 8 | R | | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Regular) - March- 2023
MATRIX ALGEBRA AND DIFFERENTIAL EQUATIONS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 60]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 10 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(10 Marks)**

1. a) Find the value of k such that the rank of matrix $\begin{pmatrix} k & -1 & 0 \\ 0 & k & -1 \\ -1 & 0 & k \end{pmatrix}$ is 2. [1M]

b) Find the rank of $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$. [1M]

c) If $A=\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -5 \\ 0 & 0 & 3 \end{pmatrix}$, then find the Eigen values of A^{-1}. [1M]

d) Find the symmetric matrix corresponding to the quadratic form $3x^2+3y^2+3z^2+4xy+8yz+8xz$. [1M]

e) State Rolle's theorem. [1M]

f) If $u=3x+y$ and $v=x-2y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$. [1M]

g) Find the integrating factor of $xy'+y=\log x$. [1M]

h) State Newton's law of cooling. [1M]

i) Solve $(D^2+4D+5)y=0$. [1M]

j) Find Particular integral of $(D^2+6D+9)y=2e^{-3x}$. [1M]

PART-B **(50 Marks)**

2. Reduce the matrix into normal form and hence find its rank $\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$. [10M]

OR

3. Find the values of 'a' and 'b' for which the equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) No solution (ii) A unique solution (iii) an Infinite number of solutions. [10M]

4. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1&2&3\\2&4&5\\3&5&6\end{pmatrix}$, Hence find A^{-1}. [10M]

OR

5. Reduce the quadratic form $2x^2+2y^2+2z^2+2yz$ to canonical form and hence find the nature, rank, index and signature of the quadratic form. [10M]

6. Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}>\cos^{-1}\left(\frac{3}{5}\right)>\frac{\pi}{3}-\frac{1}{8}$ using Lagrange's mean value theorem. [10M]

OR

7. Find the stationary points of $u(x,y)=\sin x\sin y\sin(x+y)$ where $0<x<\pi, 0<y<\pi$ and find the maximum of u. [10M]

8. Solve the differential equation $(1-x^2)\frac{dy}{dx}+xy=y^3\sin^{-1}x$. [10M]

OR

9. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hours? [10M]

10. Solve $(D^2-4D+3)y=\sin 3x\cos 2x$. [10M]

OR

11. Solve the differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$. [10M]

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