Code No.: MA101BS | **R20** | **H.T.No.** | | | 8 | R | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Supply) - January- 2022
LINEAR ALGEBRA AND CALCULUS
(Common to CSC, CSD, CSE, CSM, ECE, IT, MECH)

[Time: 3 Hours] **[Max. Marks: 70]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(20 Marks)**

1\. a) Define Rank of a matrix. [2M]

b) Find the value of k such that the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & k & 7\\ 3 & 6 & 10\end{bmatrix}$ is 2. [2M]

c) Find the eigenvalues of matrix $B=5A^2-6A+2I$, where $A=\begin{bmatrix}1 & 2 & -3\\ 0 & -1 & 2\\ 0 & 0 & -2\end{bmatrix}$. [2M]

d) Prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A. [2M]

e) Define Conditional convergence and Absolute convergence. [2M]

f) Test for convergence of $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^2}$. [2M]

g) Write the geometrical interpretation of the Rolle's theorem. [2M]

h) Evaluate $\int_0^1 x^7(1-x)^5\,dx$ [2M]

i) If $x=r\cos\theta,\ y=r\sin\theta$ then find the value of $\frac{\partial r}{\partial x}$ and $\frac{\partial \theta}{\partial y}$. [2M]

j) If $u=e^r\sec\theta;\ v=e^r\tan\theta$ then find the value of $\frac{\partial(u,v)}{\partial(r,\theta)}$. [2M]

PART-B **(50 Marks)**

2\. a). Reduce the matrix $A=\begin{bmatrix}-1 & -3 & 3 & -1\\ 1 & 1 & -1 & 0\\ 2 & -5 & 2 & -3\\ -1 & 1 & 0 & 1\end{bmatrix}$ to Echelon form and find its rank. [5M] b). Discuss for what values of λ, μ the simultaneous equations [5M]

$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ have (i) unique solution (ii) infinite number of solutions (iii) no solution.

OR

3. Solve the following system of linear equations by Gauss- Seidel iterative method correct to three decimal places [10M]

$20x+y-2z=17,\ 2x-3y+20z=25,\ 3x+20y-z=-18$

4. State Cayley- Hamilton and verify theorem for the matrix $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$. Also find A^{-1}. [10M]

OR

5. Reduce the quadratic form $2x^2+2y^2+2z^2-2yz-2zx-2xy$ to the Canonical form by orthogonal reduction and hence state nature, rank, index, and signature of the quadratic form. [10M]

6. a)Test for convergence of the series $\sum\left(\sqrt{n^3+1}-\sqrt{n^3}\right)$ [5M]

b) Test for the convergence of the series $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots$ [5M]

OR

7. Test the convergence of the series $1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\ldots\infty(x>0)$. [10M]

8. a). Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}>cos^{-1}\left(\frac{3}{5}\right)>\frac{\pi}{3}-\frac{1}{8}$ using Lagrange's mean value theorem. [5M]

b). Obtain the Taylor's series expansion of $sin\,x$ in powers of $x-\frac{\pi}{4}$. [5M]

OR

9. a).Prove that $\beta(m,n)=\frac{\Gamma m.\Gamma n}{\Gamma(m+n)}$ where $m>0,\ n>0$. [5M]

b).Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(0<b<a)$ about the major axis. [5M]

10. a).Show that the functions $u=xy+yz+zx,\ v=x^2+y^2+z^2$, and $w=x+y+z$ are functionally related. Find the relation between them. [5M]

b). If $u=sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}tan u$. [5M]

OR

11. Find the maximum volume of the rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{z^2}=1$. [10M]

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