**R18**

Code No: 151AA

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**

**B.Tech I Year I Semester Examinations, July - 2021**

**MATHEMATICS-I**

**(Common to CE, ME, ECE, EIE, MCT, MMT, ECM, AE, MIE, PTM, CSBS, CSE(AIML), CSE(IOT))**

**Time: 3 hours** **Max. Marks: 75**

**Answer any five questions**
**All questions carry equal marks**

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1. Solve the equations $8x-3y+2z=20;\ 4x+11y-z=33;\ 6x+3y+12z=35$ using Gaussian-Seidel method. [15]

2.a) Find the rank of the matrix $\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{pmatrix}$ by reducing it to Normal form.

b) Solve the system of equations by Gauss elimination method $20x+y-2z=17;\ 3x+20y-z=-18;\ 2x-3y+20z=25$. [7+8]

3.a) Determine the Eigen values and Eigen vectors of $B=2A^2-\frac{1}{2}A+3I$ where $A=\begin{pmatrix} 8 & -4 \\ 2 & 2 \end{pmatrix}$.

b) Find the nature of the quadratic form $3x^2+3y^2+3z^2+2xy+2xz-2yz$. [7+8]

4. Verify Cayley-Hamilton theorem for $A=\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and hence find $A^{-1}$ and find the Eigen values of B, where $B=A^5-4A^4-7A^3+11A^2-A-10I$. [15]

5.a) Test the series for convergence $\sum\frac{\left(n^3+1\right)^{\frac{1}{3}}-n}{\log n}$.

b) Show that the series $\frac{\cos x}{1^3}-\frac{\cos 2x}{2^3}+\frac{\cos 3x}{3^3}-\ldots$ converges absolutely. [7+8]

6.a) Show that the series $\sum(-1)^{n+1}\frac{n^3}{2^n}$ converges absolutely.

b) Show that $\int_0^\infty\frac{x^a}{a^x}dx=\frac{\Gamma(a+1)}{(\log a)^{a+1}}, if\ a>1$ [7+8]

7.a) Using Lagrange's mean value theorem, show that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}>\cos^{-1}\frac{3}{5}>\frac{\pi}{3}-\frac{1}{8}$.

b) Show that $\int_0^\infty e^{-ax}x^{n-1}dx=\frac{\Gamma(n)}{a^n}$. [8+7]

8.a) If $x=r\cos\theta, y=\sin\theta$ show that $\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}=\frac{1}{r^2}\left[\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2\right]$.

b) If $x=c\cos u\cosh v$ and $y=c\sin u\sinh v$, show that $\frac{\partial(x,y)}{\partial(u,v)}=\frac{1}{2}c^2(\cos 2u-\cosh 2v)$. [7+8]

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