

R18

Code No: 153BP

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, December - 2019

PROBABILITY AND STATISTICS AND COMPLEX VARIABLES

(Common to ME, MCT, MMT, AE, MIE, PTM)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

- 1.a) A couple has 2 children. What is the probability that both are girls if the older of the two is a girl? [2]
- b) If $p(x=2) = p(x=3)$ for a poisson variate find μ . [2]
- c) Define Null Hypothesis. [2]
- d) If $-x^3 - pxy^2 + i(3x^2y - y^3)$ is analytic find p . [2]
- e) Find the poles of $f(z) = \frac{1}{z(e^z-1)}$ [2]
- f) Two symmetric dice have both had two of their sides painted red, two painted black, one painted yellow, and the other painted white. When this pair of dice is rolled, what is the probability that both dice land with the same color face up. [3]
- g) If X is a poisson variate such that $p(x=2) = 3p(x=4)$. Find; the mean? [3]
- h) If S_1^2 and S_2^2 represent the variances of independent random samples of size $n_1 = 25$ and $n_2 = 31$ taken from normal populations with variances $\sigma_1^2 = 10$ and $\sigma_2^2 = 15$, respectively, Determine F . [3]
- i) Find the analytic function whose real part is y . [3]
- j) Calculate the Residue at the pole of $\frac{e^z}{(z-3)^3}$ [3]

PART - B

(50 Marks)

- 2.a) Suppose that A and B are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that:
 - i) either A or B occurs?
 - ii) A occurs but B does not?
 - b) An urn contains 3 red and 7 black balls. Players A and B with draw balls from the urn consecutively until a red ball is selected. Find the probability that A selects the red ball. (A draws the first ball, then B , and so on. There is no replacement of the balls drawn.) [5+5]
- OR
- 3.a) A Problem is given to three students. The probabilities of solving the problem of three students are $\frac{1}{4}, \frac{2}{5}, \frac{2}{3}$. Find the probability that the problem is solved.
 - b) Three machines produce 10%, 20%, 70% of the product in a factory. 3%, 4%, and 5% from the three machines are defective. An item is selected and found to be defective. Find the probability that it is from i) Machine-I ii) Machine-II iii) Machine III. [5+5]

4.a) The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40$ and $\sigma = 4$. What is the probability that, starting with this year, it will take over 10 years before a year occurs having a rainfall of over 50 inches?

b) Twelve percent of the population is left handed. Approximate the probability that there are at least 20 left-handers in a school of 200 students. [5+5]

OR

5.a) Find the probability of getting:

i) 2 heads ii) $P(0 \leq 3 < 5)$ iii) At least one head if 8 coins are tossed.

b) In an examination the students, who gets more than 60 will be placed in the first division, between 50 and 60 2nd division between 40 and 50 third division and less than 40 will be failed. The mean is 60% with a S.D of 10%. Find the number of students who will be placed in i) First division ii) Second division. [5+5]

6.a) An advertisement for a new toothpaste claims that it reduces cavities of children in their cavity-prone years. Cavities per year for this age group are normal with mean 3 and standard deviation 1. A study of 2,500 children who used this toothpaste found an average of 2.95 cavities per child. Assume that the standard deviation of the number of cavities of a child using this new toothpaste remains equal to 1. Are these data strong enough, at the 5 percent level of significance, to establish the claim of the toothpaste advertisement?

b) The Edison Electric Institute has published figures on the annual number of kilowatt hours expended by various home appliances. It is claimed that a vacuum cleaner expends an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners expend an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners expend, on the average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal. [5+5]

OR

7.a) Suppose that, in the past, 40% of all adults favored capital punishment. Do we have reason to believe that the proportion of adults favoring capital punishment today has increased if, in a random sample of 15 adults, 8 favor capital punishment? Use a 0.05 level of significance.

b) A soft-drink dispensing machine is said to be out of control if the variance of the contents exceeds 1.15 deciliters. If a random sample of 25 drinks from this machine has a variance of 2.03 deciliters, does this indicate at the 0.05 level of significance that the machine is out of control? Assume that the contents are approximately normally distributed. [5+5]

8.a) Let u and v denote the real and imaginary components of the function f defined by means of the equations

$$f(z) = \begin{cases} z^2 & \text{when } z \neq 0 \\ z & \text{when } z = 0 \end{cases}$$

Verify whether the Cauchy Reimann equations are satisfied at the origin.

b) Verify whether the function $g(z) = \sqrt{r} e^{i\theta}$ ($r > 0, -\pi < \theta < \pi$) is analytic in its domain of definition, if possible, find its derivative. [5+5]

OR

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- 9.a) Show that v is a harmonic conjugate of u in a domain D if and only if $-u$ is a harmonic conjugate of v in D .
- b) Find the analytic function whose imaginary part is $y^2 - x^2$. [5+5]

8R 10.a) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 1, y = \pm 1$. Evaluate $\int_C \frac{1}{3z^2+1} dz$ 8R 8R E

- b) Evaluate $\int_C \frac{dz}{(z-1)^2+3}$ where C is the positively oriented boundary of the rectangle whose sides lie along the lines $x = \pm 2, y = 0, \text{ and } y = 1$. [5+5]

OR

8R 11.a) Find the Laurent Series representation of $\frac{e^z}{(z+1)^2}$ $|z| > 1$ 8R 8R E

b) Evaluate the integral $\int_C \frac{dz}{z^3(z+4)}$ taken clockwise around the circle $C: |z+2| = 3$. [5+5]

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