Code No: 151AA **R18**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, December – 2019/January - 2020

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, PTM, ITE)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Define Hermitian, Skew-Hermitian Matrices. [2]

b) State Cayley Hamilton theorem. [2]

c) State Ratio test. [2]

d) Define Beta and Gamma functions. [2]

e) Verify the continuity of $f(x,y)=\begin{cases}\dfrac{3xy}{x^2+y^2} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$ at the origin. [2]

f) Define the rank of a matrix. [3]

g) Show that the determinant of a square matrix is equal to the product of the Eigen values for a 3×3 matrix. [3]

h) Test for the convergence of the series $\sum\left(\dfrac{n}{n+1}\right)^{n^2}$. [3]

i) Verify Rolle's mean value theorem for $f(x)=e^x(\sin x-\cos x)$ in $\left[\dfrac{\pi}{4},\dfrac{5\pi}{4}\right]$. [3]

j) If $z=f(x+ay)+g(x-ay)$ prove that $\dfrac{\partial^2 z}{\partial y^2}=a^2\dfrac{\partial^2 z}{\partial x^2}$. [3]

PART- B

(50 Marks)

2.a) Find the rank of $\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$ by Normal form.

b) Find whether the following system of equations are consistant if so solve them $x-y+2z=5,\ 2x+y-z=1,\ 3x+y+z=8$. [5+5]

OR

3. Solve the following system of linear equations by using Gauss-Seidel method

$$20x+y-2z=17$$

$$3x+20y-z=-18$$

$$2x-3y+20z=25$$

[10]

4. Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [10]

OR

5.a) Find the rank, index, signature of the quadratic form $x^2 - 2y^2 + 3z^2 - 4yz + 6zx$.

b) Find the nature of the quadratic form $2x^2 + 2y^2 + 2z^2 + 2yz$. [5+5]

6.a) Test whether the series is conditionally convergent or absolutely convergent $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \ldots$.

b) Examine the convergence of the series $\sum \frac{x^n}{n!}$. [5+5]

OR

7.a) Examine the absolute convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$.

b) Test the convergence of the series $\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots\ldots$

[5+5]

8.a) Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$.

b) Verify Rolle's theorem for $f(x) = x(x+3)e^{-\frac{x}{2}}$ in $[-3, 0]$. [5+5]

OR

9.a) Verify Cauchy's mean value theorem for x^2 and $\frac{1}{x^2}$ in (2, 4).

b) Prove that $\frac{\beta(p,q)}{p+q} = \frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p}$ $(p, q > 0)$ [5+5]

10.a) Show that the function $f(x,y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither a maximum nor a minimum at $(0,0)$.

b) If $x = r\cos\theta$ and $y = r\sin\theta$, show that $\frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{r}$. [5+5]

OR

11.a) If $u = f(r)$, where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.

b) Find the area of a greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

[5+5]

---ooOoo---