# Bridges

The bridges are used for not only the measurement of resistances but also used for the measurement of various components like capacitance, inductance etc.

A bridge circuit in its simplest form consists of a network of four resistance arms forming a closed circuit. A source of current is applied to two opposite junctions. The current detector is connected to other two junctions.

The bridges circuit use the 'comparison measurement' methods & operate on 'null indication principle'. The bridge circuit compares the value of an unknown component with that of an accurately known standard component. Thus the accuracy depends on the bridge components & not on the null indicator. Hence high degree of accuracy can be obtained.

In a bridge circuit, when no current flows through the null detector which is generally galvanometer, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called 'balancing condition or balancing equation'. This equation gives us the value of the unknown component.

## Advantages of bridge circuit

1. The balance equation is independent of the magnitude of the input voltage or its source impedance. These quantities do not appear in the balance equation.
2. the measurement accuracy is high as the measurement is done by comparing the unknown value with standard value.
3. The accuracy is independent of the characteristics of a null detector & is dependent on the component values.
4. the balancing equation is independent of the sensitivity of the null detector, the impedance of the detector or any impedance shunting the detector.
5. the balancing condition remains unchanged if the source & detector are interchanged.
6. The bridge circuits can be used in the control circuits. when used in such control applications one arm of the bridge contains a resistive element that is sensitive to the physical parameter like pressure, temperature etc. which is to be controlled.

## Types of bridges

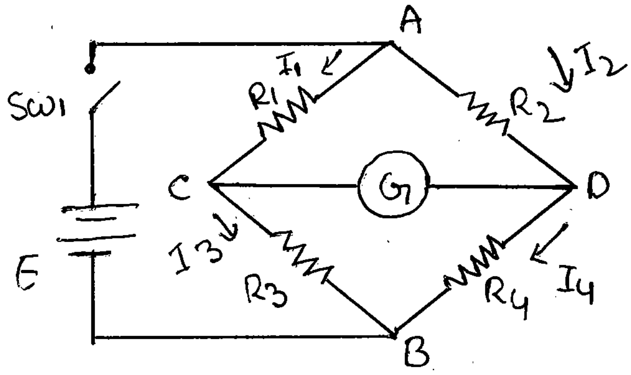
The two types of bridges are

1. DC bridges   2. AC bridges

The d.c. bridges are used to measure the resistances while the a.c bridges are used to measure the impedances consisting capacitances & inductances. The d.c bridges use the dc voltage as the excitation voltage while the a.c bridges use the alternating voltage as the exicitation voltage.

# Wheat Stone's Bridge

Wheat stone's bridge is the most accurate method available for measuring resistances. This bridge circuit is used to determine resistance ranging from approximately an ohm ($1\Omega$) to several ohms ($M\Omega$)




The source of emf (battery) & switch is connected to points A & B, while a sensitive current indicating meter, the galvanometer, is connected to points C & D. The galvanometer is a sensitive microammeter with a zero centre scale. When there is no current through the meter, the pointer rests at '0' i.e midscale. Current in one direction causes the pointer to deflect on one side & current in opposite direction to the other side.

When SW1 is closed, current flows & divides into the two arms at point A ie $I_1$ & $I_2$. The bridge is balanced when there is no current through the galvanometer or when the potential difference at points C & D is equal i.e the potential across the galvanometer is zero.

To obtain the bridge balance equation, we have from fig

$$I_1 R_1 = I_2 R_2 \longrightarrow (1)$$

For the galvanometer current to be zero, the following conditions should be satisfied

$$I_1 = I_3 = \frac{E}{R_1+R_3}$$

$$I_2 = I_4 = \frac{E}{R_2+R_4}$$

Substituting in equation (1) we get

$$\frac{E \times R_1}{R_1+R_3} = \frac{E \times R_2}{R_2+R_4}$$

$$R_1(R_2+R_4) = (R_1+R_3)R_2$$

$$R_1R_2 + R_1R_4 = R_1R_2 + R_2R_3$$

$$R_4 = \frac{R_2R_3}{R_1}$$

This is the equation for the bridge to be balanced.

In a practical wheatstones bridge atleast one of the resistance is made adjustable, to permit balancing. when the bridge is balanced, the unknown resistance (normally connected at $R_4$) may be determined from the setting of the adjustable resistor, which is called standard resistor

Hence $R_x = \frac{R_2R_3}{R_1}$

## Sensitivity of wheat stone Bridge

when the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection. The amount of deflection depends on the sensitivity of the galvanometer. This senstivity can be expressed as amount of deflection per unit current.

$$\text{senstivity } S = \frac{\text{deflection } D}{\text{current } I} \quad mm/\mu A;\ radians/\mu A;\ degrees/\mu A.$$

Another way of representing the galvanometer senstivity is the amount of deflection per unit voltage across the galvanometer. This called voltage senstivity of the galvanometer

$$S_V = \frac{\theta}{e} \quad d/v \quad r/v$$

e = voltage across galvanometer
θ = Deflection of galvanometer

while the bridge senstivity is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance

$$S_B = \frac{\theta}{\Delta R/R}$$

$\Delta R/R$ = unit fractional change in unknown resistance.

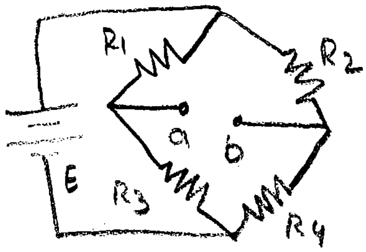
## Wheatstone bridge under small unbalance

The bridge sensitivity can be calculated by solving the bridge for small unbalance.

To determine the amount of deflection that would result for a particular degree of unbalance, general circuit analysis can be applied, but we shall use Thevenin's theorem.

Since we are intrested in determining the current through the galvanometer, we wish to find the Thevenin's equivalent, as seen by the galvanometer

Thevenin's equivalent voltage is found by disconnecting the galvanometer from the bridge & determine the open circuit voltages between terminals a & b



unbalanced bridge

Applying the voltage divider equation, the voltage at point a can be determined as follows

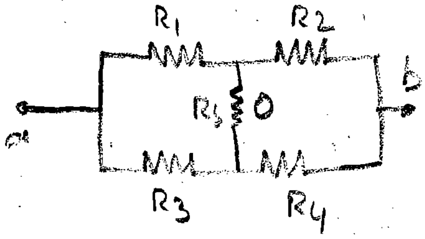
$$E_a = \frac{E \times R_3}{R_1 + R_3}$$ at point b, $$E_b = \frac{E \times R_4}{R_2 + R_4}$$

Therefore the voltage between a & b is the difference between $E_a$ & $E_b$ which represents Thevenin's equivalent voltage.

therefore $E_{th} = E_{ab} = E_a - E_b = \dfrac{E \times R_3}{R_1 + R_3} - \dfrac{E \times R_4}{R_2 + R_4}$

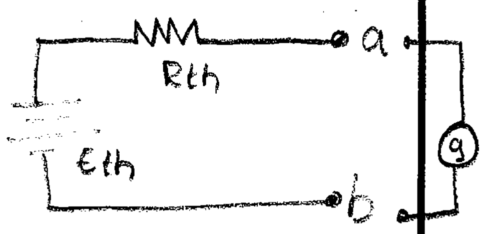
$$E_{ab} = E_{th} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

Thevenin's equivalent resistance can be determined by replacing the voltage source E with its internal impedance or otherwise short-circuited & calculating the resistance looking into terminals a & b. Since the internal resistance is assumed to be very low, we treat it as 0Ω. Thevenin's equivalent resistance of the circuit is $R_1 \parallel R_3$ in series with $R_2 \parallel R_4$ i.e $R_1 \parallel R_3 + R_2 \parallel R_4$



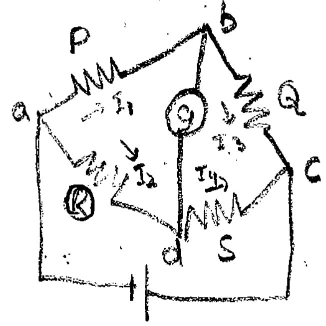

$$\therefore R_{Th} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$

If a galvanometer is connected across terminal a & b; it will expereince the same deflectional the output of the bridge. The magnitude of current is limited by both Thevenin's equivalent resistance & any resistance connected between a & b. The resistance b/w a & b consists only of the galvanometer resistance Rg. The deflection current in the galvanometer is therefore given by




$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

prob: In the wheat stone bridge, the values of resistances of various arms are $P = 1000\Omega$, $Q = 100\Omega$, $R = 2{,}005\Omega$ & $S = 200\Omega$. The battery has an emf of 5V & negligible internal resistance. The galvanometer has a current sensitivity of 10 mm/μA & internal resistance of 100Ω calculate the deflection of galvanometer & senstivity of the bridge interms of deflection per unit change in resistance.




Resistance of unknown resistor required for balance $R = \left(P/Q\right) S = \frac{PS}{Q} = \frac{1000 \times 200}{100} = 2000\Omega$

In the actual bridge the unknown resistor has a value of 2005Ω or the deviation from the balance conditions is $\Delta R = 2005 - 2000 = 5\Omega$

Thevenins source generator $E_{th} = E\left[\frac{R}{R+S} - \frac{P}{P+Q}\right]$

$$= 5\left[\frac{2005}{2005+200} - \frac{1000}{1000+100}\right] = 1.0307 \times 10^{-3} V$$

Internal resistance of bridge looking into terminals b&d

$$R_{th} = \frac{RS}{R+S} + \frac{PQ}{P+Q} = 272.8\Omega$$

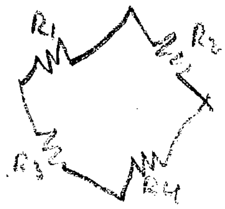
Hence the current through galvanometer

$$I_g = \frac{E_o}{R_o + G} = \frac{1.0307 \times 10^{-3}}{272.8 + 100} A = 2.77\mu A$$

Deflection of galvanometer $\theta = S_i I_g = 10 \times 2.77 = 27.7\,mm$

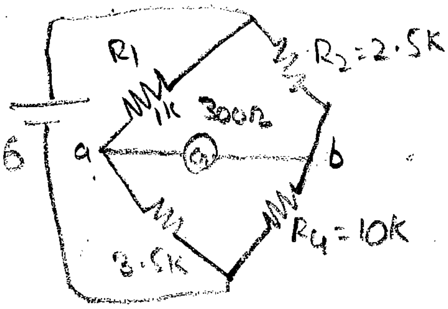
Senstivity of bridge $S_B = \frac{\theta}{\Delta R} = \frac{27.7}{5} = 5.54\,mm/\Omega$

prob: Fig consists of following $R_1 = 10k$, $R_2 = 15K$ & $R_3 = 40k$ Find the unknown resistance $R_x$.




From balancing eq $R_x = \frac{R_2 R_3}{R_1} = 60k\Omega$

prob: A unbalance wheatstone bridge is given in fig. calculate the current through the galvanometer




The Thevenins equivalent voltage between a & b $E_{th} = E\left[\frac{R_4}{R_2+R_4} - \frac{R_3}{R_1+R_3}\right]$

$E_{th} = 0.132V$

Thevenin's equivalent resistance is $R_{th} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$

$R_{th} = 2.778k$

The current through the galvanometer is given by

$$I_g = \frac{E_{th}}{R_{th} + R_g} = \frac{0.132V}{2.788k + 0.3k} = 42.88\,\mu A$$

Advantages:

1. The result are not dependent on the calibration & characteristics of galvanometer & as it works on null deflection

2. The source emf & inaccuracies due to the source fluctuations do not affect the balance of bridge. Hence the corresponding errors are completely avoided.

3. Due to null deflection method used, the accuracy & senstivity is higher than direct deflection meters.

Limitations:

For low resistance measurement, the resistance of the leads & contacts becomes significant & Introduces an error.

The bridge cannot be used for high resistance measurements This is because while such measurement the resistance presented by the bridge becomes so large that the galvanometer becomes insensitive to show any imbalance

Similarly heating effect due to large current also plays a major role. the excessive currents may generate heat which may cause the permanent change in resistance

Applications:

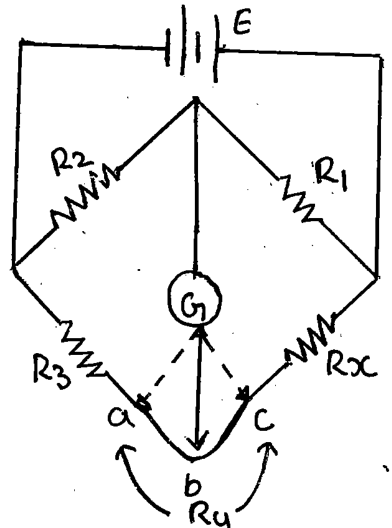
It is used to measure the dc resistance of various types of wires for the purpose of quality control of wire.

It is used to measure the resistance of motor winding, relay

# Kelvin's Bridge:

when the resistance to be measured is of the order of magnitude of bridge contact & lead resistance, a modified form of wheatstone's bridge, the kelvins bridge is employed.

kelvin's bridge is a modification of wheatstone's bridge & is used to measure values of resistance below 1Ω. In low resistance measurement, the resistance of the lead connecting the unknown resistance to the terminal of the bridge circuit may affect the measurement.




consider the circuit where $R_y$ represents the resistance of the connecting leads from $R_3$ to $R_x$. The galvanometer can be connected either to point c or point a

when it is connected to point a, the resistance $R_y$ of the connecting lead is added to the unknown resistance $R_x$. when the connection is made to point c, $R_y$ is added to the bridge arm $R_3$ & resulting measurement of $R_x$ is lower than the actual value, because now the actual value of $R_3$ is higher than its nominal value by the resistance $R_y$. If the galvanometer is connected to point b in between c & a in such way that the ratio of the resistance from c to b & that from a to b equals the ratio of resistances $R_1$ & $R_2$ then

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \longrightarrow (1)$$

the usual balance equations for the bridge give the relationship

$$R_x + R_{cb} = \frac{R_1}{R_2}(R_3 + R_{ab}) \longrightarrow (2)$$

but $R_{ab} + R_{cb} = R_y$ & $\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$

adding '1' on both sides

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

i.e $\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$

$\therefore R_{ab} = \frac{R_2 R_y}{R_1 + R_2}$ & as $R_{ab} + R_{cb} = R_y$

$$R_{cb} = R_y - R_{ab} = R_y - \frac{R_2 R_y}{R_1 + R_2}$$

$$\therefore R_{cb} = \frac{R_1 R_y}{R_1 + R_2} \Leftarrow R_{cb} = \frac{R_1 R_y + R_2 R_y - R_2 R_y}{R_1 + R_2}$$

substituting for $R_{ab}$ & $R_{cb}$ in equation (2)

$$R_x + \frac{R_1 R_y}{R_1+R_2} = \frac{R_1}{R_2}\left[R_3 + \frac{R_2 R_y}{R_1+R_2}\right]$$

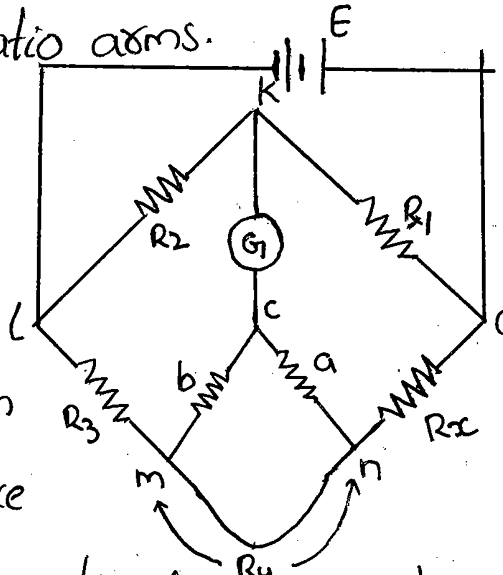
$$R_x + \frac{R_1 R_y}{R_1+R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_2 R_y}{R_2(R_1+R_2)}$$

Hence $R_x = \frac{R_1 R_3}{R_2}$

The obtained equation is the usual wheatstones balance equation & it indicates that the effect of the resistance of the connecting leads from point a to point c has been eliminated by connecting the galvanometer to an intermediate position b.

## Kelvins Double bridge:

The above principle forms the basis of the construction of kelvins double bridge. It is a double bridge because it incorporates a second set of ratio arms.




The second set of arms a & b connect the galvanometer to a point c at the appropriate potential between m & n connection i.e $R_y$. The ratio of the resistance of arms a & b is the same as the ratio of $R_1$ & $R_2$. The galvanometer indication is zero when potentials at K & C are equal.

$$\therefore \; E_{lk} = E_{lmc}$$

But $E_{lk} = \dfrac{R_2}{R_1 + R_2} \times E$ & $E = I\left[R_3 + R_x + \dfrac{(a+b)R_y}{a+b+R_y}\right]$

consider the path from l-m-n-o bak to l through battery E. The resistance b/w the terminals m & n is parallel combination of Ry & (a+b)

$E = I \times \left[R_3 + R_y \,||\, (a+b) + R_x\right]$

we get $E_{lk} = \dfrac{R_2}{R_1 + R_2} \times I\left[R_3 + R_x + \dfrac{(a+b)R_y}{a+b+R_y}\right]$

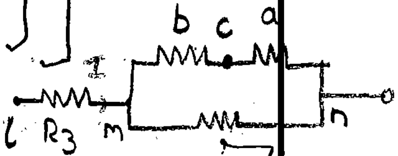
$V_{ln} = I \times \left[\dfrac{R_y(a+b)}{R_y + a + b}\right]$

$V_{lc} = \dfrac{b}{a+b} \cdot V_{ln}$

$V_{lmc} = IR_3 + V_{lc}$

Similarly $E_{lmc} = I\left[R_3 + \dfrac{b}{a+b}\left[\dfrac{(a+b)R_y}{a+b+R_y}\right]\right]$

$E_{lmc}$ consider path from terminal l to n →




But $E_{lk} = E_{lmc}$

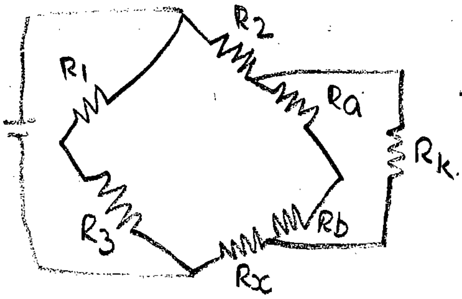
$$\frac{IR_2}{R_1 + R_2}\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] = I\left[R_3 + \frac{b}{a+b}\left\{\frac{(a+b)R_y}{a+b+R_y}\right\}\right]$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{(a+b+R_y)}\left[\frac{R_1}{R_2} - \frac{a}{b}\right]$$

but $\dfrac{R_1}{R_2} = \dfrac{a}{b}$

$$\therefore \; R_x = \frac{R_1 R_3}{R_2}$$

It indicate that the resistance of the connecting lead Ry has no effect on the measurement; provided that the ratios of the resistance of the two sets of ratio arms are equal. In a typical kelvins bridge the range of resistance covered is 1 – 0.00001Ω (10 Mohm) prob: If in fig the ratio of Ra to Rb is 1000Ω, $R_1$ is 5Ω & $R_1 = 0.5R_2$ what is the value of Rx




Sol $\frac{Rx}{R_2} = \frac{Ra}{Rb} = \frac{1}{1000}$

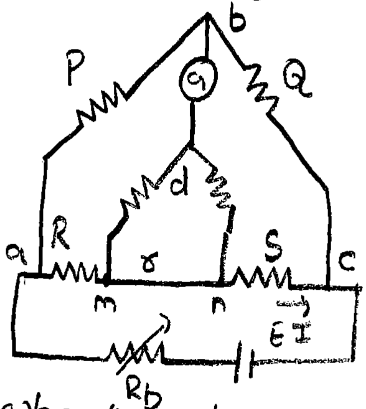
Since $R_1 = 0.5R_2$ ∴ $R_2 = 5/0.5 = 10Ω$

∴ $Rx/10 = 1/1000 = 0.01Ω$

prob: A Kelvin double bridge each of the ratio arms P = Q = p = q = 1000Ω. The emf of the battery is 100V & a resistance of 5Ω is included in the battery circuit. The galvanometer has a resistance of 500Ω & the resistance of the link connecting the unknown resistance to the standard resistance may be neglected. The bridge is balanced when the standard resistance S = 0.001Ω

a. Determine the value of unknown resistance

b. Determine the current through the unknown resistance R at balance




c. Determine the deflection of galvanometer when the unknown resistance R is changed by 0.1% from its value at balance. The galvanometer has a sensitivity of 200 mm/MA

At balance, the value of unknown resistance

$$R = \frac{PS}{Q} = \frac{1000 \times 0.1}{1000} = 0.001Ω$$

b. If we examine the kelvin bridge ckt, we find the resistor P, Q & P, q are in parallel with the resistance of link r. Since r is negligible & P, Q ; P & q have large values ; the effect of ratio arms can be neglected for the purpose of calculation of current.

$$I = \frac{E}{R_b + R + S} = \frac{100}{5 + 0.001 + 0.0001} \simeq 20A$$

c. The value R is changed by 0.1 percent

∴ New value $R = 0.0001 \times 0.001 = 0.000001\Omega$

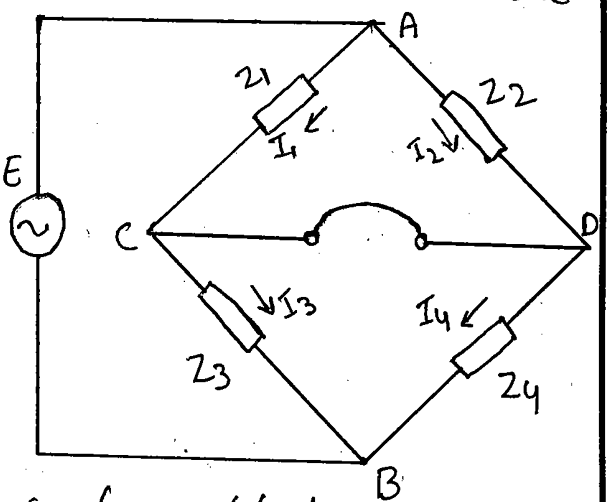
Voltage b/w point a & c

$$E_{ad} = \frac{R + S + r}{R_b + R + S + r} \cdot E$$

**AC Bridges:** An a.c bridge in its basic form consists of four arms, a source of excitation & a balanced detector. Each arm consists of an impedance. The source is an a.c supply supplies a.c voltage at the required frequency.

For high frequencies, the electronic oscillators are used as the source. The balance detectors commonly used for a.c bridges are headphones, tunable amplifiers or vibration galvanometers. Balancing of AC bridges is more difficult than DC bridges because both magnitude & phase angles condition have to be satisfied.

The simple ac bridge is the outcome of the wheatstone bridge.




Bridge Balance Equation

For bridge balance, the potential of point C must be same as the potential of D. These potential must be equal in terms of amplitude as well as phase. Thus the drop from A to C must be equal to drop across A to D, in both magnitude & phase for the bridge balance

$$\therefore \bar{E}_{AC} = \bar{E}_{AD}$$

The vector indicates both amplitude & phase to be considered

$$\therefore \overline{I_1 Z_1} = \overline{I_2 Z_2} \longrightarrow (1)$$

when the bridge is balanced, no current flows through the headphones.

$\therefore \ \bar{I_3} = \bar{I_1} \ \ \& \ \ \bar{I_4} = \bar{I_2}$

Now $\bar{I_1} = \dfrac{\bar{E}}{\bar{Z_1}+\bar{Z_3}} \rightarrow (2) \ \ \& \ \ \bar{I_2} = \dfrac{\bar{E}}{\bar{Z_2}+\bar{Z_4}} \rightarrow (3)$

substitute 2 & 3 in equation 1

$$\frac{\bar{E}\cdot\bar{Z_1}}{\bar{Z_1}+\bar{Z_3}} = \frac{\bar{E}\cdot\bar{Z_2}}{\bar{Z_2}+\bar{Z_4}}$$

$\therefore \ \overline{Z_1 Z_4} = \overline{Z_2 Z_3}$ → balance equ in impedance form

In the admittance form $\overline{Y_1 Y_4} = \overline{Y_2 Y_3}$

Now in the polar form the impedances are expressed as

$\bar{Z_1} = Z_1 \angle\theta_1$ ; $\bar{Z_2} = Z_2 \angle\theta_2$ ; $\bar{Z_3} = Z_3 \angle\theta_3$ ; $\bar{Z_4} = Z_4 \angle\theta_4$

Note that the product of the impedances must be carried out in polar form where magnitudes get multiplied & phase angles get added.

$$Z_1 \angle\theta_1 \times Z_4 \angle\theta_4 = Z_2 \angle\theta_2 \times Z_3 \angle\theta_4$$

$$Z_1 Z_4 \angle\theta_1 + \angle\theta_4 = Z_2 Z_3 \angle\theta_2 + \theta_3$$

Equating magnitudes of both sides $Z_1 Z_4 = Z_2 Z_3$

Equating phase angles $\theta_1 + \theta_4 = \theta_2 + \theta_3$

For inductive branch, $Z_L = R + jX_L = |Z_L| \angle +\theta$

$Z_C = R - jX_C = |Z_C| \angle -\theta$

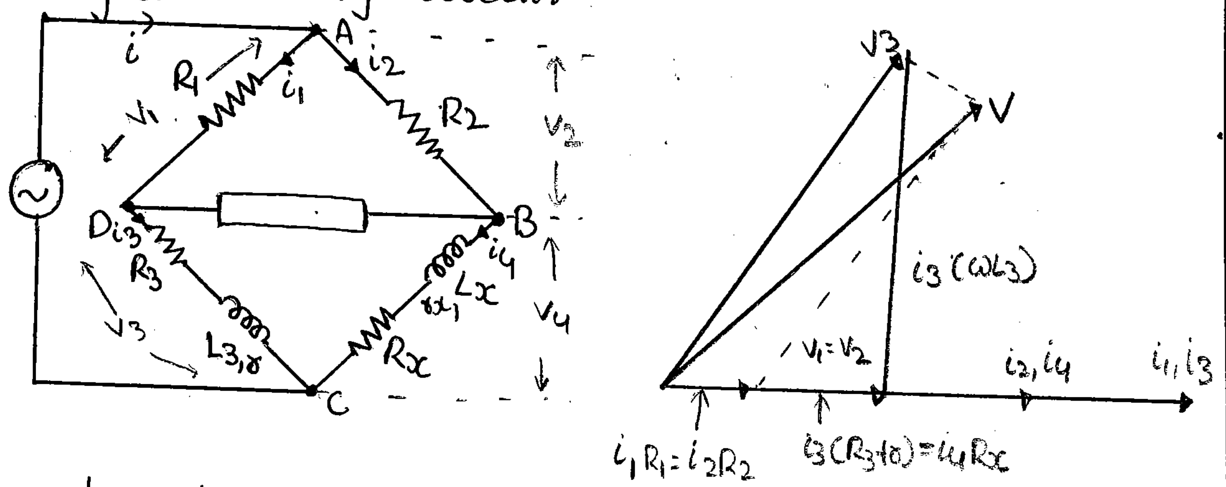
$X_L = 2\pi f L \ \Omega$ & $X_C = \dfrac{1}{2\pi f C} \Omega$

# Maxwell's Bridge

Maxwells bridge can be used to measure inductance by comparison either with a variable standard self inductance or with a standard variable capacitance.

## Maxwell's Inductance Bridge

using this bridge, we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit




Two branches consist of non. inductive resistances $R_1$ & $R_2$ one of the arms consists variable inductance with series resistance $R_3$. The remaining arm consists unknown inductance $L_x$ & unknown resistance $R_x$

At balance, we get condition as

$$\frac{R_1}{[(R_3+r)+j\omega L_3]} = \frac{R_2}{(R_x+r_x)+j\omega L_x}$$

$$R_1\left[(R_x+r_x)+j\omega L_x\right] = R_2\left[(R_3+r)+j\omega L_3\right]$$

$$\therefore\ R_1R_x + R_1r_x + j\omega R_1L_x = R_2(R_3+r) + j\omega R_2L_3$$

$$\Rightarrow\ R_1(R_x+r_x) + j\omega R_1L_x = R_2(R_3+r) + j\omega R_2L_3$$

Equating imaginary terms

$$R_1L_x = R_2L_3$$

$$\boxed{L_x = \frac{R_2}{R_1}L_3}$$

$$R_1R_x = \frac{R_2(R_3+r) - R_1r}{R_1}$$

$$= \frac{R_2}{R_1}(R_3+r) - r_x$$

Equating real terms

$$R_1(R_x+r_x) = R_2(R_3+r)$$

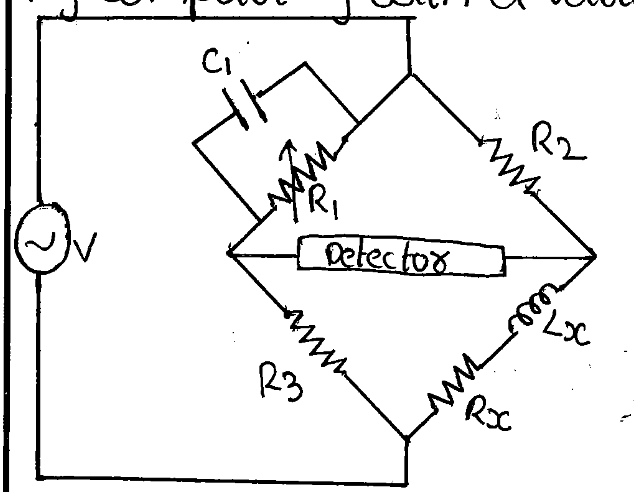
$$(R_x+r_x) = \frac{R_2}{R_1}(R_3+r)$$

without $r$ & $r_x$

$$L_x = \frac{R_2L_3}{R_1}\ ;\ R_x = \frac{R_2R_3}{R_1}$$

Maxwells Inductance Capacitance Bridge

using this bridge, we can measure inductance by comparing with a variable standard capacitor.



Maxwell's Bridge measures an unknown inductance interms of a known capacitance. The use of standard arm offers the advantage of compactness & easy shielding. The capacitor is almost a loss-less component. one arm has a resistance $R_1$ in parallel with $C_1$ & hence it is easier to write the balance equation using the admittance of arm1 instead of impedance.

The general equation of bridge balance is $Z_1 Z_4 = Z_2 Z_3 \Rightarrow Z_1 Z_x = Z_2 Z_3$

i.e $Z_x = \dfrac{Z_2 Z_3}{Z_1}$ $\quad\therefore Z_x = Z_2 Z_3 Y_1 \longrightarrow (1)$

where $Z_1 = R_1$ in parallel with $C_1$ i.e $Y_1 = \dfrac{1}{Z_1}$

$Z_2 = R_2$ ; $Z_3 = R_3$ $\qquad Y_1 = \dfrac{1}{R_1} + j\omega C_1$

$Z_x = R_x$ inseries with $L_x$ $= R_x + j\omega L_x$

From equation (1)

$$R_x + j\omega L_x = R_2 R_3 \left[ \frac{1}{R_1} + j\omega C_1 \right]$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real parts

$$R_x = \frac{R_2 R_3}{R_1}$$

Equating imaginary parts

$$L_x = C_1 R_2 R_3$$

Also Q factor $= \frac{\omega L_x}{R_x} = \omega C_1 R_1$

This bridge is also called Maxwell wien bridge

Advantages:

1. The balance equation is independent of losses associated with inductance
2. the balance equation is independent of frequency of measurement.
3. The scale of resistance can be calibrated to read the inductance directly
4. The scale of $R_1$ can be calibrated to read Q value directly
5. when the bridge is balanced, the only component in series with coil undertest is resistance $R_2$. If $R_2$ is selected such that it can carry high current, then heavy current carrying capacitance y coils can be tested using this bridge.

Dis advantages:

1. It cannot be used for the measurement of high Q values. Its use is limited to the measurement of low Q values from 1 to 10. this can be proved from phase angle balance condition which says that sum of the angles of one pair of opposite arms must be equal

$\theta_1 + \theta_4 = \theta_2 + \theta_3$

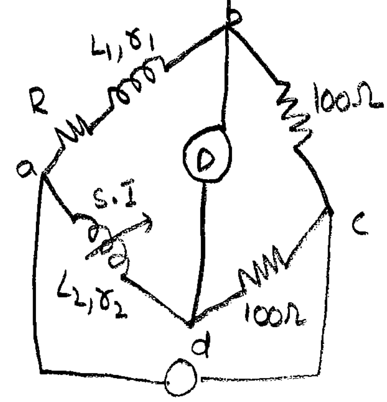
But $\theta_2$ & $\theta_3$ are zero as the corresponding impedances are pure resistances. For high Q values, the angle $\theta_4$ is almost $90°$. Hence $\theta_1$ must be $-90°$. But $\theta_1$ get decided by parallel combination of $R_1$ & $C_1$. To get $\theta_1$ as almost $-90°$, the value of $R_1$ should be very very high. practically such high resistance is not possible. Hence high Q values cannot be measured.

2. There is an interaction between the resistance & reactances balances. Getting the balance adjustment is little difficult.

3. It is unsuited for the coils with low Q values, less than one, because of balance convergence problem.

Prob: A maxwell's inductance comparison bridge is shown in fig; Arm ab consists of a coil with inductance $L_1$ & resistance $r_1$ in series with a non inductive resistance R. Arm bc & ad are each a non-inductive resistance of $100\Omega$. Arm ad consists of standard variable inductor of resistance ~~32.7Ω~~ $32.7\Omega$. Balance is obtained when $L_2 = 47.8mH$ & $R = 1.36\Omega$. Find the resistance & inductance of the coil in arm ab

At balance




$$[(R_1 + r_1) + j\omega L_1] \times 100 = (r_2 + j\omega L_2) \times 100$$

Equating real & imaginary

$$R_1 + r_1 = r_2 \quad \& \quad L_2 = L_1$$

$\therefore$ Resistance of coil $r_1 = r_2 - R_1 = 32.7 - 1.36 = 31.34\Omega$

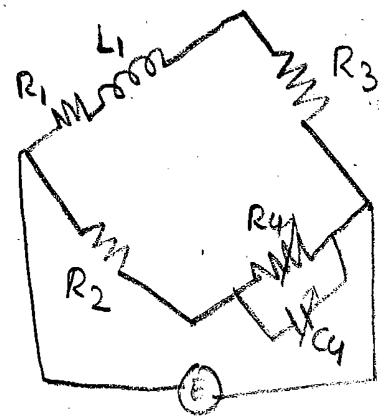
$\therefore$ Inductance $L_1 = L_2 = 47.8mH$

prob: A maxwell's capacitance bridge shown in fig is used to measure an unknown inductance in comparison with capacitance. The various values at balance

$R_2 = 400\Omega$ ; $R_3 = 600\Omega$ ; $R_4 = 1000\Omega$ ; $C_4 = 0.5\mu F$

Calculate the values of $R_1$ & $L_1$. Calculate also the value of ~~rang~~. storage (Q) factor of coil if frequency is 1000 Hz

At balance




$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2 R_3$$

Seperating real & imaginary

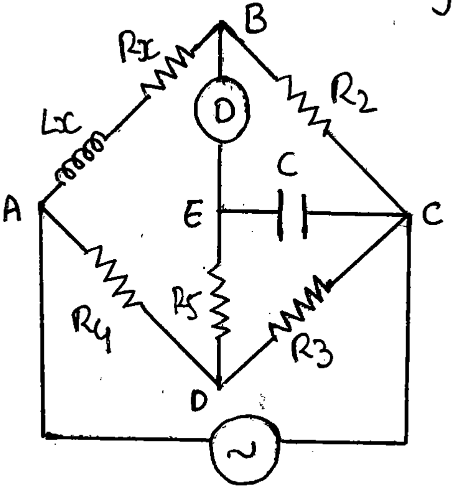
$$R_1 = \frac{R_2 R_3}{R_4} = \frac{400 \times 600}{1000} = 240\Omega$$

$$L_1 = R_2 R_3 C_4 = 400 \times 600 \times 0.5 \times 10^{-6} = 0.12H$$

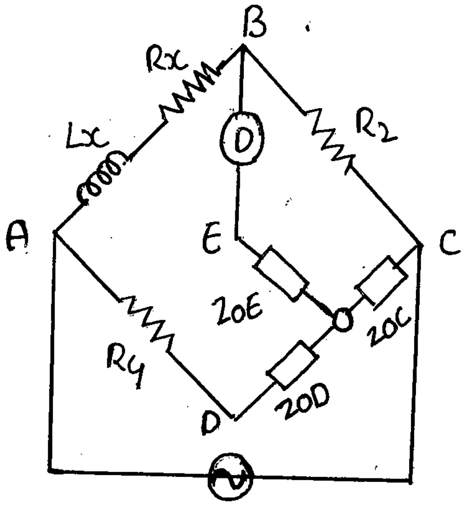
storage factor $Q = \frac{\omega L_1}{R_1} = \frac{2\pi \times 1000 \times 0.12}{240} =$

# Andersons bridge

In the Andersson's bridge, the unknown inductor's value is determined in terms of known values of capacitance & resistance. This is a modified form of maxwell weins bridge.

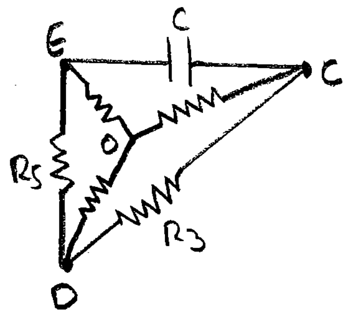



a.




b.

The balancing condition of this bridge can be easily obtained by converting the mesh impedance (delta) C, R5, R3 to a equivalent star with the star point O as shown in fig b by using star/delta transformation. As per delta to star transformation.




$$Z_{OD} = \frac{R_3 R_5}{R_3 + R_5 + 1/j\omega C}$$

$$Z_{OC} = \frac{R_3 / j\omega C}{R_3 + R_5 + 1/j\omega C} = Z_3$$

Hence with fig b

$$Z_1 = R_x + j\omega L_x \quad Z_2 = R_2 \quad Z_3 = Z_{OC} = \frac{R_3 / j\omega C}{R_3 + R_5 + 1/j\omega C} \quad Z_4 = R_4 + Z_{OD}$$

For balance condition

$$Z_1 Z_3 = Z_2 Z_4$$

$$(R_x + j\omega L_x)\left[\frac{R_3/j\omega C}{R_3 + R_5 + 1/j\omega C}\right] = R_2\left[R_4 + \frac{R_3 R_5}{R_3 + R_5 + 1/j\omega C}\right]$$

$$[R_x + j\omega L_x]\frac{R_3}{j\omega C} = R_2 R_4 (R_3 + R_5 + 1/j\omega C) + R_2 R_3 R_5$$

$$\frac{R_x R_3}{j\omega C} + \frac{j\omega L_x R_3}{j\omega C} = R_2 R_4 R_3 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

$$-\frac{j R_x R_3}{\omega C} + \frac{L_x R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 - \frac{j R_2 R_4}{\omega C} + R_2 R_3 R_5$$

Equating the real & imaginary parts

$$\frac{L_x R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5$$

$$L_x = \frac{C}{R_3}\left[R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5\right]$$

$$L_x = C R_2 \left[R_4 + \frac{R_4 R_5}{R_3} + R_5\right]$$

$$\frac{-jR_xR_3}{\omega C} = \frac{-jR_2R_4}{\omega C}$$

$$R_xR_3 = R_2R_4$$

$$R_x = \frac{R_2R_4}{R_3}$$

If the capacitor used is not perfect; the value of inductance remains unchanged, but the value of $R_x$ changes

This method can also be used to measure the capacitance of the capacitor c if a calibrated self inductance is available.

Advantages

1. can be used for accurate measurement of capacitance interms of inductance
2. other bridges require variable capacitor but a fixed capacitor can be used for anderson's bridge
3. The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q

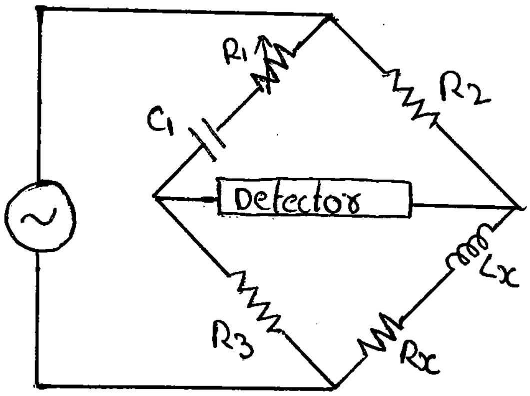
Disadvantages

1. It is more complicated than other bridges
2. uses more number of components
3. Balance equations are also complicated to derive
4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

# Hay's bridge

The limitation of Maxwell's bridge is that it cannot be used for high Q values. The Hay's bridge is suitable for the coils having high Q values.

The difference in Maxwell's bridge & Hay's bridge is that the Hay's bridge consists of resistance $R_1$ in series with the standard capacitor $C_1$ in one of the ratio arms. Hence for larger phase angles $R_1$ needed is very low, which is practicable. Hence bridge can be used for the coils with high Q values.




At balance $Z_1 Z_x = Z_2 Z_3$ where

$Z_1 = R_1 - j/\omega C_1$ ; $Z_2 = R_2$ ; $Z_3 = R_3$; $Z_x R_x + j\omega L_x$

$$\left[R_1 - j/\omega C_1\right]\left[R_x + j\omega L_x\right] = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$$

Equating real & imaginary

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \& \quad \frac{R_x}{\omega C_1} = \omega L_x R_1$$

Solving for $L_x$ & $R_x$ we have

$$R_x = \omega^2 L_x C_1 R_1 \rightarrow (1)$$

$$\Rightarrow R_1 \left[\omega^2 L_x C_1 R_1\right] + \frac{L_x}{C_1} = R_2 R_3$$

$$\omega^2 R_1^2 C_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

$$L_x + \omega^2 R_1^2 C_1^2 L_x = R_2 R_3 C_1$$

$$\therefore \boxed{L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1^2}}$$

substitute $L_x$ in equation (1)

$$\boxed{R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 R_1^2 C_1^2}}$$

$$Q = \frac{1}{\omega C_1 R_1}$$

The term $\omega$ appears in the expression for both $L_x$ & $R_x$

This indicates that the bridge is frequency sensitive

# Advantages of Hays Bridge

1. It is best suitable for the measurement of inductance with high Q, typically greater than 10
2. It gives very simple expression for Q factor interms of elements in the bridge
3. It requires very low value Resistor $R_1$ to measure high Q inductance.

# Disadvantage

It is only suitable for measurement of high Q inductance Consider expression for unknown inductance $L_x = \frac{R_2 R_3 C_1}{1+\left[\frac{1}{Q}\right]^2}$

For high Q inductances, $(1/Q^2)$ term can be neglected But for low Q measurements, $(1/Q^2)$ term is significant, hence cannot be neglected. Hence Hays bridge is not suitable for the measurement of low Q inductances.

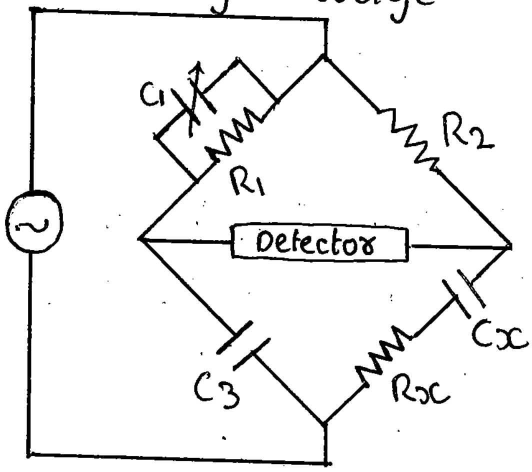
prob: Find the series equivalent inductance & resistance of the network that causes Hay's bridge to null with the following bridges.

$R_3 = 1k\Omega$ $\omega = \underset{\text{angular freq}}{3000}$ rad/s, $R_2 = 10k\Omega$, $R_1 = 2k\Omega$, $C_1 = 1\mu F$

$$R_x = \frac{\omega^2 R_1 R_2 R_3 C_1^2}{1+\omega^2 R_1^2 C_1^2} = 4.86k\Omega \; ; \; L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1^2} = 270mH$$

# Schering bridge

A very important bridge used for the precision measurement of capacitors & their insulating properties is Schering's bridge.




The standard capacitor $C_3$ is standard air capacitor having very stable value.

For balance, the general equation is

$$Z_1 Z_x = Z_2 Z_3 \Rightarrow Z_2 Z_3 Y_1 = Z_x$$

where $Z_x = R_x - j/\omega C_x$ ; $Z_2 = R_2$ ; $Z_3 = -j/\omega C_3$ ; $Y_1 = \frac{1}{R_1} + j\omega C_1$

$$\left(R_x - j/\omega C_x\right) = R_2 \left(-j/\omega C_3\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$= \frac{R_2(-j)}{R_1[\omega C_3]} + \frac{R_2 C_1}{C_3}$$

Equating real & imaginary

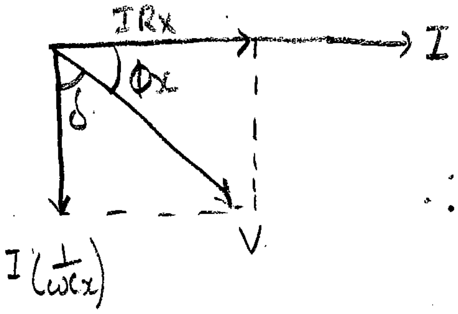
$$R_x = \frac{R_2 C_1}{C_3} \quad \& \quad C_x = \frac{R_1 C_3}{R_2}$$

**power factor:** The powerfactor of the series RC combination is defined as the cosine of the phase angle of the circuit

$$p.f = \cos\phi_x = \frac{R_x}{Z_x}.$$

For phase angles very close to 90°, the reactance is almost equal to impedance $\therefore$ $p.f = \frac{R_x}{X_x} = \frac{R_x}{\left(\frac{1}{\omega C_x}\right)} = \omega R_x C_x$

**Loss angle:** For a series combination of $R_x$ & $C_x$, the angle between the voltage across the series combination & voltage across the capacitor $C_x$ is called loss angle $\delta$




From fig $\tan\delta = \frac{IR_x}{I\left(\frac{1}{\omega C_x}\right)} = \omega R_x C_x$

$$\therefore \cancel{\tan\delta = \omega}\left(\underset{\frac{R_2C_1}{C_3}}{\frac{R_3C_4}{C_2}}\right)\left(\underset{\frac{R_1C_3}{R_2}}{\cancel{\frac{R_4C_2}{R_3}}}\right) = \omega R_4 C_4$$

$$\therefore \tan\delta = \omega\left(\frac{R_2C_1}{C_3}\right)\left(\frac{R_1C_3}{R_2}\right) = \omega R_1 C_1$$

**Dissipation factor (D):** For $R_x$-$C_x$ series circuit, it is cotangent of the phaseangle $\phi_x$.

$$D = \cot\phi_x = \frac{1}{\tan\phi_x} = \frac{1}{\left[\frac{I\left(\frac{1}{\omega C_x}\right)}{IR_x}\right]} = \omega R_x C_x = \omega R_1 C_1$$

The quality factor Q is reciprocal of dissipation factor

$$Q = \frac{1}{\omega C R} \quad \left[D = \frac{1}{Q}\right]$$

Sources of Errors in Bridge circuits.

we have assumed in our derivation of basic circuit relationships that a bridge consists of lumped impedance units connected only by the wires that are placed in the circuit for making connections. This idealized condition exists in bridge circuits to a lesser or greater extent. the idealized arrangment works fairly well if the frequency is low, if component impedances are not high & if accuracy desired is not high. But in practice there are certain factors which we have not considered yet & these complicate the behaviour or bridge circuits

Some of these factors are

Stray couplings between one bridge arm & another & from elements to ground. these stray couplings modify the balance conditions making a definite balance impossible or may lead to false balance.

Stray conductance effects due to imperfect insulation

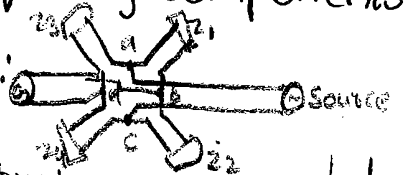
mutual inductance effects due to magnetic coupling between various components of the bridge

Stray capacitance effects due to electrostatic fields between conductors at different potentials.

residues in components. e.g the existence of small amount of series, inductance or shunt capacitance in nominally non-reactive resistors.

Techniques used for Reducing Errors: (or) Sawhney

1. use of high quality components : high accuracy, minimum of residuels
2. Bridge layout:




3. Sensitivity
   bridge must be operated at sufficient sensitivity
4. Stray conductance Effects : The various bridge components & other pieces of apparatus may be mounted on insulating stands
5. Eddy current effects. Errors may result due to variation in the value of standards, which may occur because of induced eddy currents in the standard resistors & inductors. In order to avoid such errors the presence of large conducting masses near the bridge network is avoided.

precautions To be taken when using a bridge

The leads should be carefully laid out in such a way that no loops or long lengths enclousing magnetic flux are produced with consequent stray inductance errors.

with a large L, the self capacitance of the leads is more important than their inductance, so they should be spaced relatively far apart.

In very precise inductive & capacitances measurements leads are encased in metal tubes to shield them from mutual electromagnetic action.